Universidade Federal de Uberlândia
Engenharia Eletrônica e de Telecomunicações

- Processamento digital de sinais –
Capítulo 4 – Transformada discreta de Fourier

Prof. Alan Petrônio Pinheiro

O que veremos

1) Introdução
2) Entendendo a equação da DFT
3) Simetria da DFT
4) Linearidade e magnitude da DFT
5) Eixo da frequência
6) Inversa da DFT
7) Leakage/”vazamento”
8) Janelamento de sinais
9) Resolução e preenchimento com zeros
10) Representações no domínio da frequência
11) A FFT

1) Introdução

- A importância da DFT no PDS

- Origem

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

tempo $\Longleftrightarrow$ frequência

- Correspondente discreta:

$$X(m) = \sum_{n-0}^{N-1} x(n) e^{-j2\pi nm/N}$$

2) Entendendo a equação da DFT

- Reescrevendo:

$$X(m) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nm/N}$$

$$X(m) = \sum_{n=0}^{N-1} x(n) \left[\cos\left(2\pi \frac{nm}{N} \right) - j.sen\left(2\pi \frac{nm}{N} \right) \right]$$

- onde *N* = número de amostras a serem processadas

= número de pontos no eixo da frequência

- *n* varia de 0 até N-1

- Exemplo 1: considere quando N=4 e Fs=500

$$X(m) = \sum_{n=0}^{N-1} x(n)\left[\cos\left(2\pi\frac{nm}{N}\right) - j.sen\left(2\pi\frac{nm}{N}\right)\right]$$

$$X(m) = \sum_{n=0}^{3} x(n)\left[\cos\left(2\pi\frac{nm}{4}\right) - j.sen\left(2\pi\frac{nm}{4}\right)\right]$$

$$\begin{aligned} X(0) = {} & x(0)\cos\left(2\pi\frac{0.0}{4}\right) - j.x(0).sen\left(2\pi\frac{0.0}{4}\right) \\ & + x(1)\cos\left(2\pi\frac{1.0}{4}\right) - j.x(1).sen\left(2\pi\frac{1.0}{4}\right) \\ & + x(2)\cos\left(2\pi\frac{2.0}{4}\right) - j.x(2).sen\left(2\pi\frac{2.0}{4}\right) \\ & + x(3)\cos\left(2\pi\frac{3.0}{4}\right) - j.x(3).sen\left(2\pi\frac{3.0}{4}\right) \end{aligned}$$

$$\begin{aligned} X(1) = {} & x(0)\cos\left(2\pi\frac{0.1}{4}\right) - j.x(0).sen\left(2\pi\frac{0.1}{4}\right) \\ & + x(1)\cos\left(2\pi\frac{1.1}{4}\right) - j.x(1).sen\left(2\pi\frac{1.1}{4}\right) \\ & + x(2)\cos\left(2\pi\frac{2.1}{4}\right) - j.x(2).sen\left(2\pi\frac{2.1}{4}\right) \\ & + x(3)\cos\left(2\pi\frac{3.1}{4}\right) - j.x(3).sen\left(2\pi\frac{3.1}{4}\right) \end{aligned}$$

Vale destacar que:

$$f(m) = m.\frac{Fs}{N}$$

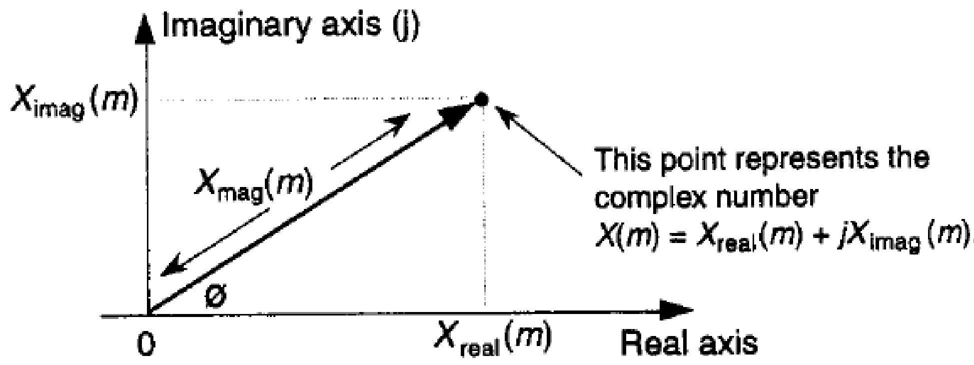
Assim:

$$X(0) = 0.\frac{500}{4} = 0Hz$$ (primeiro termo de frequência – DC)

$$X(1) = 1.\frac{500}{4} = 125Hz$$ (segundo termo de frequência)

$$X(2) = 2.\frac{500}{4} = 250Hz$$ (terceiro termo de frequência)

$$X(3) = 3.\frac{500}{4} = 375Hz$$ (quarto termo de frequência)



$$X(m) = X_{real}(m) + jX_{imag}(m) = X_{mag}(m) \text{ no ângulo de } X_{\phi}(m)$$

- A magnitude é:

$$X_{mag}(m) = |X(m)| = \sqrt{X_{real}(m)^2 + X_{imag}(m)^2}$$

- O ângulo é

$$X_{\phi}(m) = \tan^{-1}\left(\frac{X_{imag}(m)}{X_{real}(m)}\right)$$

- A potência do espectro é:

$$X_{PS}(m) = X_{mag}(m)^2 = X_{real}(m)^2 + X_{imag}(m)^2$$

- Exemplo 2: calcule as oito primeiras componentes de frequência da DFT do sinal considerando F_s=8000 a/s

$$x(t) = sen(2\pi 1000t) + 0.5sen(2\pi 2000t + 3\pi / 4)$$

Solução:

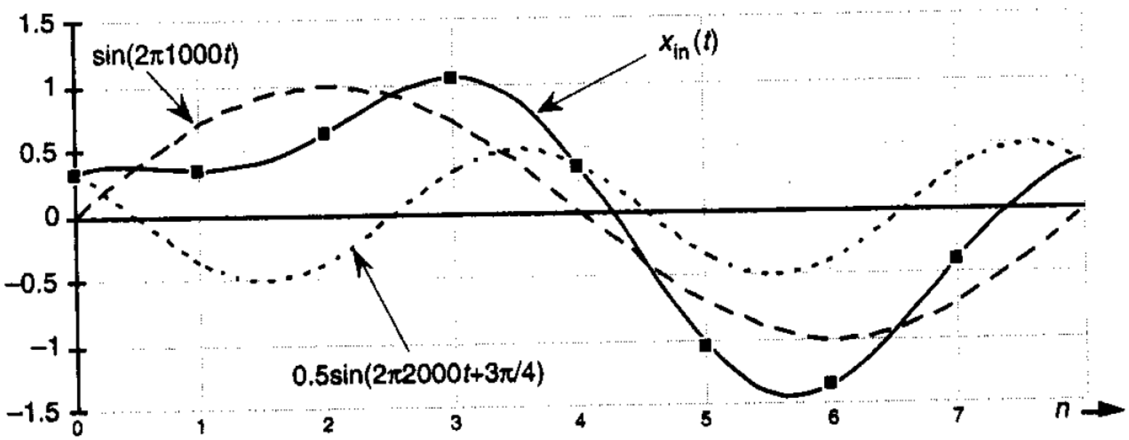
Discretizando:

$$x(n) = sen(2\pi 1000nt_s) + 0.5sen(2\pi 2000nt_s + 3\pi / 4)$$

As componentes de frequência serão: 0Hz, 1kHz, 2kHz,... 7kHz e o sinal será:

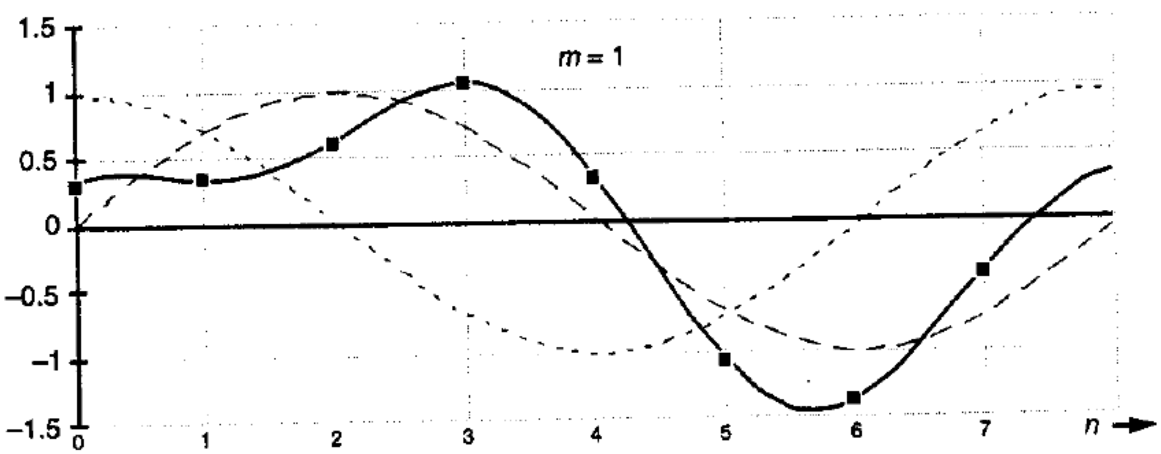
$$x(0) = 0.3535, \quad x(1) = 0.3535,$$
$$x(2) = 0.6464, \quad x(3) = 1.0607,$$
$$x(4) = 0.3535, \quad x(5) = -1.0607,$$
$$x(6) = -1.3535, \quad x(7) = -0.3535$$

O formato do sinal é:



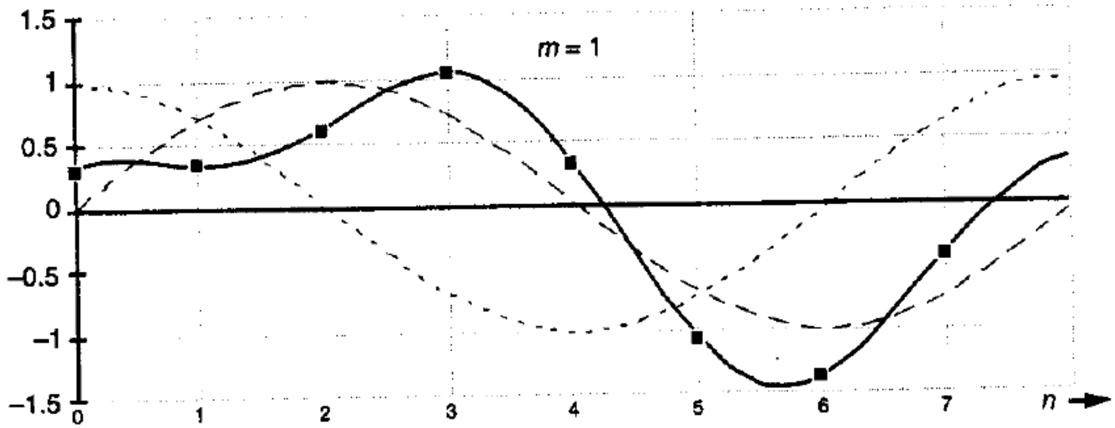
A primeira componente é:

$$X(1) = \sum_{N=0}^{7} x(n)\cos(2\pi n/8) - jx(n)sen(2\pi n/8)$$



$$
\begin{aligned}
X(1) = {} & 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) && \leftarrow \text{this is the } n = 0 \text{ term} \\
& + 0.3535 \cdot 0.707 && - j(0.3535 \cdot 0.707) && \leftarrow \text{this is the } n = 1 \text{ term} \\
& + 0.6464 \cdot 0.0 && - j(0.6464 \cdot 1.0) && \leftarrow \text{this is the } n = 2 \text{ term} \\
& + 1.0607 \cdot -0.707 && - j(1.0607 \cdot 0.707) && \quad \cdots \\
& + 0.3535 \cdot -1.0 && - j(0.3535 \cdot 0.0) && \quad \cdots \\
& - 1.0607 \cdot -0.707 && - j(-1.0607 \cdot -0.707) && \quad \cdots \\
& - 1.3535 \cdot 0.0 && - j(-1.3535 \cdot -1.0) && \quad \cdots \\
& - 0.3535 \cdot 0.707 && - j(-0.3535 \cdot -0.707) && \leftarrow \text{this is the } n = 7 \text{ term}
\end{aligned}
$$

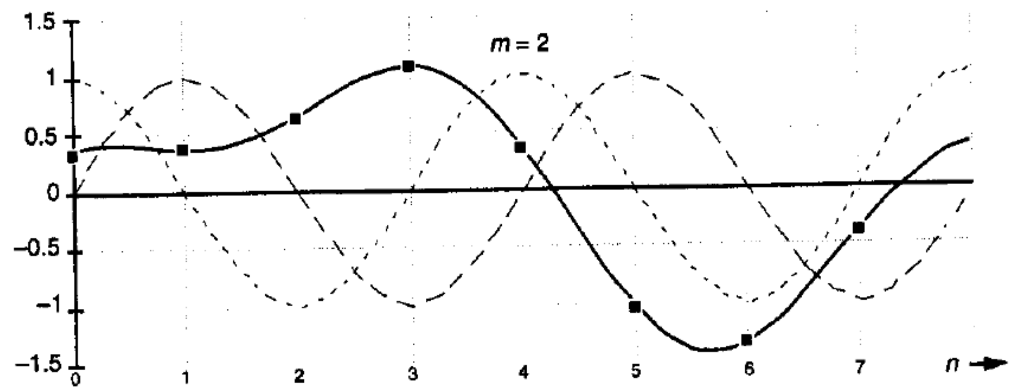
$$
\begin{aligned}
= {} & 0.3535 && + j0.0 \\
& + 0.250 && - j0.250 \\
& + 0.0 && - j0.6464 \\
& - 0.750 && - j0.750 \\
& - 0.3535 && - j0.0 \\
& + 0.750 && - j0.750 \\
& + 0.0 && - j1.3535 \\
& - 0.250 && - j0.250
\end{aligned}
$$



$$= 0.0 - j4.0 = 4 \angle -90^\circ.$$

$$
\begin{aligned}
X(2) = {} & 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& + 0.3535 \cdot 0.0 && - j(0.3535 \cdot 1.0) \\
& + 0.6464 \cdot -1.0 && - j(0.6464 \cdot 0.0) \\
& + 1.0607 \cdot 0.0 && - j(1.0607 \cdot -1.0) \\
& + 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& -1.0607 \cdot 0.0 && - j(-1.0607 \cdot 1.0) \\
& -1.3535 \cdot -1.0 && - j(-1.3535 \cdot 0.0) \\
& -0.3535 \cdot 0.0 && - j(-0.3535 \cdot -1.0)
\end{aligned}
$$

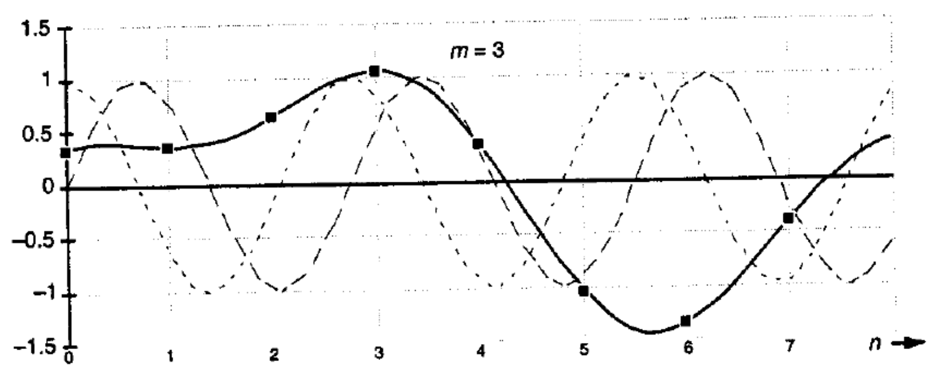
$$
\begin{aligned}
= {} & 0.3535 && + j0.0 \\
& + 0.0 && - j0.3535 \\
& - 0.6464 && - j0.0 \\
& - 0.0 && + j1.0607 \\
& + 0.3535 && - j0.0 \\
& + 0.0 && + j1.0607 \\
& + 1.3535 && - j0.0 \\
& - 0.0 && - j0.3535
\end{aligned}
$$



$$= 1.414 + j1.414 = 2 \angle 45^\circ.$$

$$
\begin{aligned}
X(3) = &\ 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
&+ 0.3535 \cdot -0.707 && - j(0.3535 \cdot 0.707) \\
&+ 0.6464 \cdot 0.0 && - j(0.6464 \cdot -1.0) \\
&+ 1.0607 \cdot 0.707 && - j(1.0607 \cdot 0.707) \\
&+ 0.3535 \cdot -1.0 && - j(0.3535 \cdot 0.0) \\
&- 1.0607 \cdot 0.707 && - j(-1.0607 \cdot -0.707) \\
&- 1.3535 \cdot 0.0 && - j(-1.3535 \cdot 1.0) \\
&- 0.3535 \cdot -0.707 && - j(-0.3535 \cdot -0.707)
\end{aligned}
$$

$$
\begin{aligned}
= &\ 0.3535 && + j0.0 \\
&- 0.250 && - j0.250 \\
&+ 0.0 && + j0.6464 \\
&+ 0.750 && - j0.750 \\
&- 0.3535 && - j0.0 \\
&- 0.750 && - j0.750 \\
&+ 0.0 && + j1.3535 \\
&+ 0.250 && - j0.250
\end{aligned}
$$

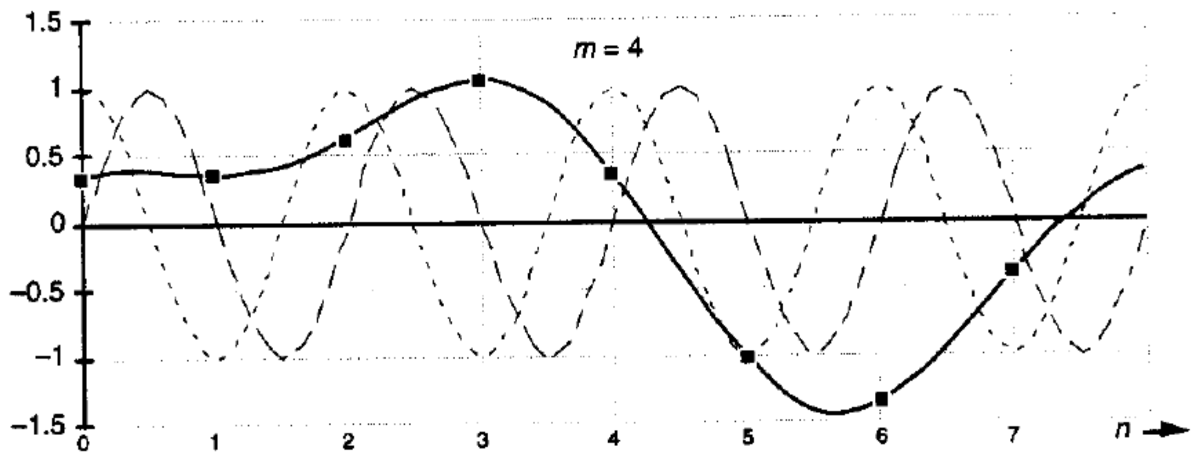


$$= 0.0 - j0.0 = 0 \angle 0°.$$

$$
\begin{aligned}
X(4) = {} & 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& + 0.3535 \cdot -1.0 && - j(0.3535 \cdot 0.0) \\
& + 0.6464 \cdot 1.0 && - j(0.6464 \cdot 0.0) \\
& + 1.0607 \cdot -1.0 && - j(1.0607 \cdot 0.0) \\
& + 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& - 1.0607 \cdot -1.0 && - j(-1.0607 \cdot 0.0) \\
& - 1.3535 \cdot 1.0 && - j(-1.3535 \cdot 0.0) \\
& - 0.3535 \cdot -1.0 && - j(-0.3535 \cdot 0.0)
\end{aligned}
$$

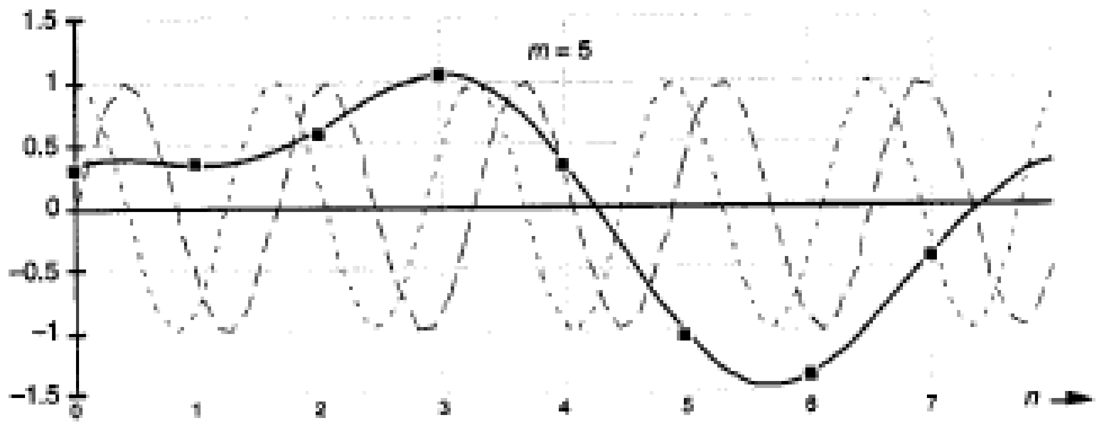
$$
\begin{aligned}
= {} & 0.3535 && - j0.0 \\
& - 0.3535 && - j0.0 \\
& + 0.6464 && - j0.0 \\
& - 1.0607 && - j0.0 \\
& + 0.3535 && - j0.0 \\
& + 1.0607 && - j0.0 \\
& - 1.3535 && - j0.0 \\
& + 0.3535 && - j0.0
\end{aligned}
$$

$$= 0.0 - j0.0 = 0 \angle 0^\circ.$$



$$
\begin{aligned}
X(5) = \ & 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& + 0.3535 \cdot -0.707 && - j(0.3535 \cdot -0.707) \\
& + 0.6464 \cdot 0.0 && - j(0.6464 \cdot 1.0) \\
& + 1.0607 \cdot 0.707 && - j(1.0607 \cdot -0.707) \\
& + 0.3535 \cdot -1.0 && - j(0.3535 \cdot 0.0) \\
& - 1.0607 \cdot 0.707 && - j(-1.0607 \cdot 0.707) \\
& - 1.3535 \cdot 0.0 && - j(-1.3535 \cdot -1.0) \\
& - 0.3535 \cdot -0.707 && - j(-0.3535 \cdot 0.707)
\end{aligned}
$$

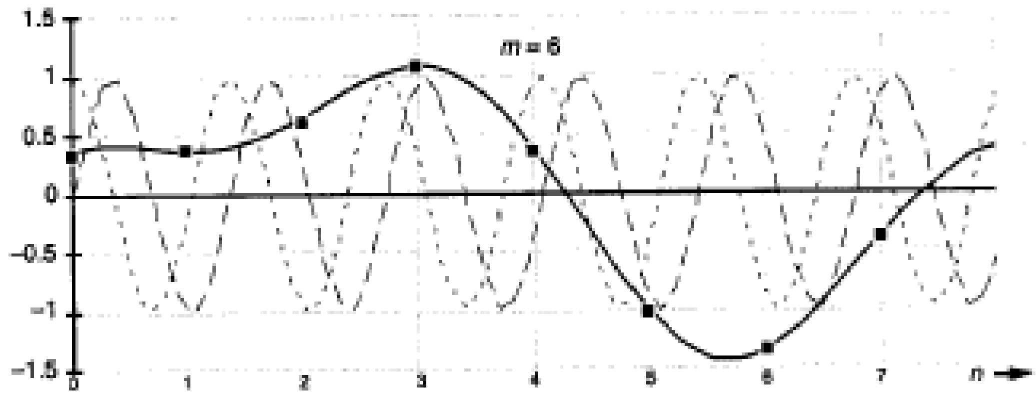
$$
\begin{aligned}
= \ & 0.3535 && - j0.0 \\
& - 0.250 && + j0.250 \\
& + 0.0 && - j0.6464 \\
& + 0.750 && + j0.750 \\
& - 0.3535 && - j0.0 \\
& - 0.750 && + j0.750 \\
& + 0.0 && - j1.3535 \\
& + 0.250 && + j0.250
\end{aligned}
$$



$$= 0.0 - j.0 = 0 \angle 0^\circ.$$

$$
\begin{aligned}
X(6) = & \; 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& + 0.3535 \cdot 0.0 && - j(0.3535 \cdot -1.0) \\
& + 0.6464 \cdot -1.0 && - j(0.6464 \cdot 0.0) \\
& + 1.0607 \cdot 0.0 && - j(1.0607 \cdot 1.0) \\
& + 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& - 1.0607 \cdot 0.0 && - j(-1.0607 \cdot -1.0) \\
& - 1.3535 \cdot -1.0 && - j(-1.3535 \cdot 0.0) \\
& - 0.3535 \cdot 0.0 && - j(-0.3535 \cdot 1.0)
\end{aligned}
$$

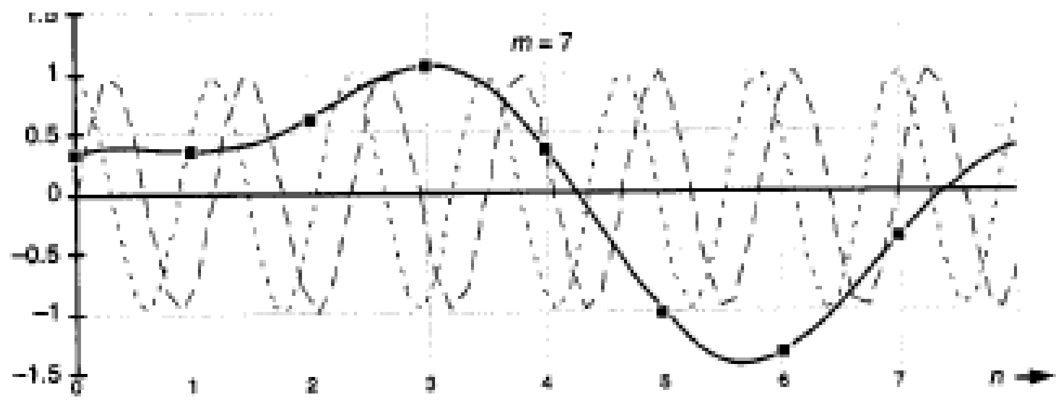
$$
\begin{aligned}
= \; & 0.3535 && - j0.0 \\
& + 0.0 && + j0.3535 \\
& - 0.6464 && - j0.0 \\
& + 0.0 && - j1.0607 \\
& + 0.3535 && - j0.0 \\
& + 0.0 && - j1.0607 \\
& + 1.3535 && - j0.0 \\
& + 0.0 && + j0.3535
\end{aligned}
$$



$$= 1.414 - j1.414 = 2 \angle -45^\circ.$$

$$
\begin{aligned}
X(7) = {} & 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& + 0.3535 \cdot 0.707 && - j(0.3535 \cdot -0.707) \\
& + 0.6464 \cdot 0.0 && - j(0.6464 \cdot -1.0) \\
& + 1.0607 \cdot -0.707 && - j(1.0607 \cdot -0.707) \\
& + 0.3535 \cdot -1.0 && - j(0.3535 \cdot 0.0) \\
& - 1.0607 \cdot -0.707 && - j(-1.0607 \cdot 0.707) \\
& - 1.3535 \cdot 0.0 && - j(-1.3535 \cdot 1.0) \\
& - 0.3535 \cdot 0.707 && - j(-0.3535 \cdot 0.707)
\end{aligned}
$$

$$
\begin{aligned}
= {} & 0.3535 && + j0.0 \\
& + 0.250 && + j0.250 \\
& + 0.0 && + j0.6464 \\
& - 0.750 && + j0.750 \\
& - 0.3535 && - j0.0 \\
& + 0.750 && + j0.750 \\
& + 0.0 && + j1.3535 \\
& - 0.250 && + j0.250
\end{aligned}
$$



$$= 0.0 + j4.0 = 4 \angle 90^\circ.$$

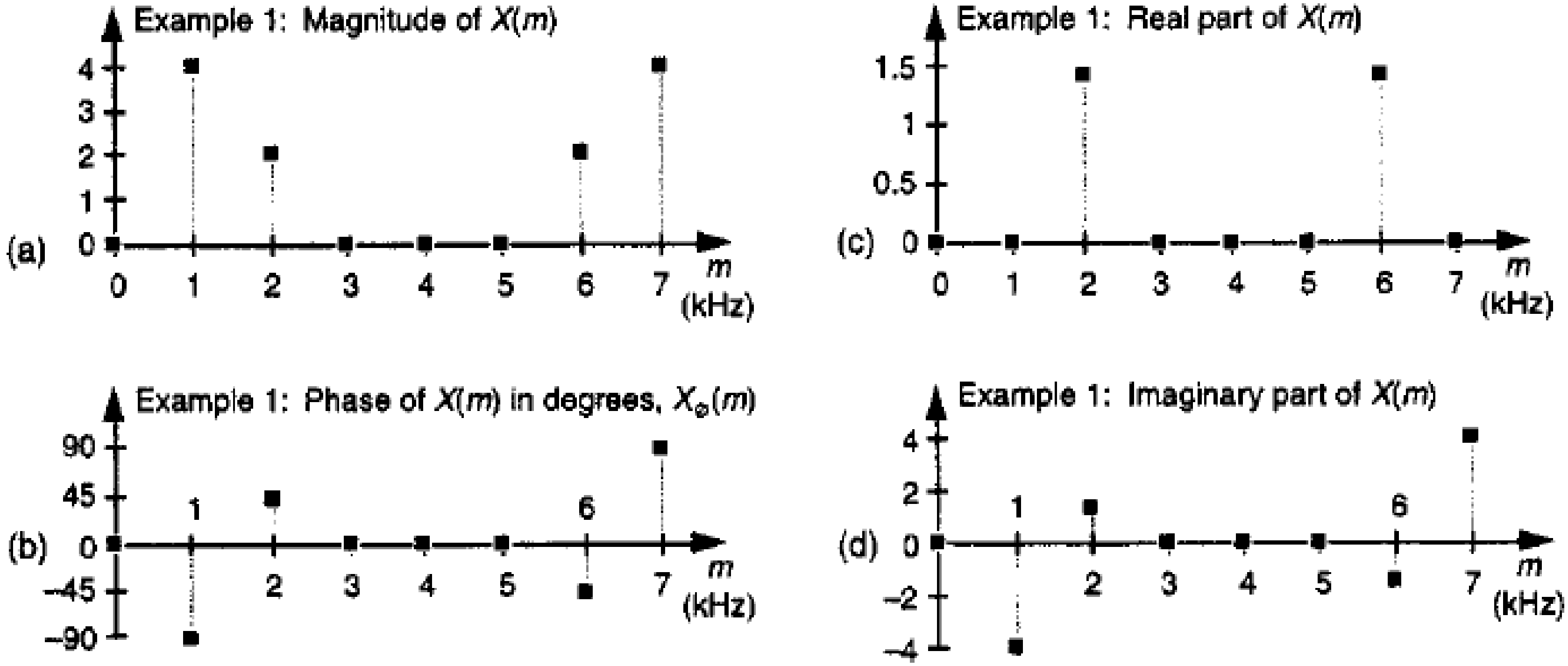
$$X(0) = \sum_{n=0}^{N-1} x(n)[\cos(0) - j\sin(0)] .$$

$$X(0) = \sum_{n=0}^{N-1} x(n) .$$

$$\begin{aligned}
X(0) = {} & 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& + 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& + 0.6464 \cdot 1.0 && - j(0.6464 \cdot 0.0) \\
& + 1.0607 \cdot 1.0 && - j(1.0607 \cdot 0.0) \\
& + 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& - 1.0607 \cdot 1.0 && - j(-1.0607 \cdot 0.0) \\
& - 1.3535 \cdot 1.0 && - j(-1.3535 \cdot 0.0) \\
& - 0.3535 \cdot 1.0 && - j(-0.3535 \cdot 0.0)
\end{aligned}$$

$$\begin{aligned}
X(0) = {} & 0.3535 && - j0.0 \\
& + 0.3535 && - j0.0 \\
& + 0.6464 && - j0.0 \\
& + 1.0607 && - j0.0 \\
& + 0.3535 && - j0.0 \\
& - 1.0607 && - j0.0 \\
& - 1.3535 && - j0.0 \\
& - 0.3535 && - j0.0
\end{aligned}$$

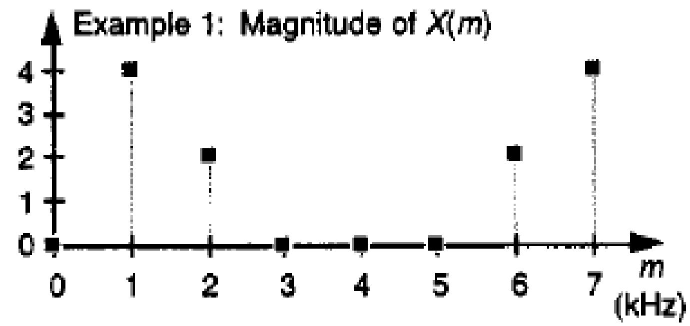
$$= 0.0 - j0.0 = 0 \angle 0^\circ .$$



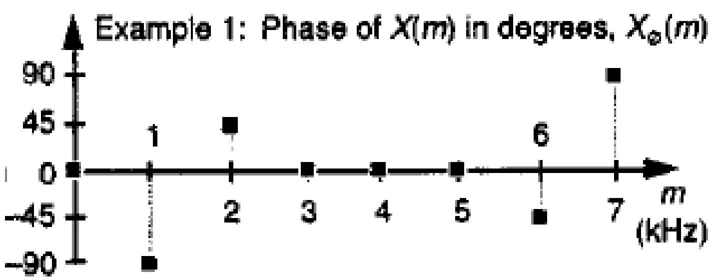
- A fase é relativa ao cosseno!
  - Ex.: -90° equivale a cos(α-90°)=sin(α)

3) Simetria

- Forma geral: $X(m) = X^*(N-m)$
- Simetria par da magnitude



- Simetria ímpar da fase (complexo conjugado)



- Conclusão: informação relevante só até N/2 -1

- Prova matemática simetria:

$$X(N-m)=\sum_{n=0}^{N-1}x(n)e^{-j2\pi n(N-m)/N}=\sum_{n=0}^{N-1}x(n)e^{-j2\pi nN/N}e^{-j2\pi n(-m)/N}$$

$$=\sum_{n=0}^{N-1}x(n)e^{-j2\pi n}e^{j2\pi nm/N}$$

Lembrando de conjugado complexo:

$x = a + jb$
$x^* = a - jb$

$x = e^{j\alpha}$
$x^* = e^{-j\alpha}$

Sendo que $e^{-j2\pi n}=\cos(2\pi n)-j\sin(2\pi n)=1$

$$X(N-m)=\sum_{n=0}^{N-1}x(n)e^{j2\pi nm/N}$$

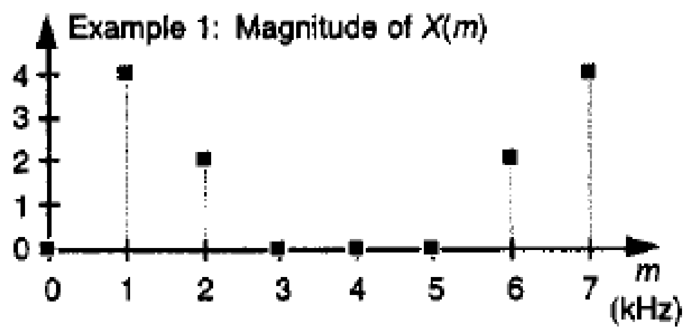
$$X^*(N-m)=\sum_{n=0}^{N-1}x(n)e^{-j2\pi nm/N}=X(m)$$

4) Linearidade e magnitude da DFT

- Linearidade: $x_{soma}(n) = x_1(n) + x_2(n)$

$$X_{soma}(m) = X_1(m) + X_2(m)$$

- Magnitude:
  - Ex.: $x(t) = sen(2\pi 1000t) + 0.5sen(2\pi 2000t + 3\pi/4)$



  - "Solução":

$$X(m) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi nm/N}$$

5) Eixo da frequência

- Resolução frequência:

$$f_{resol.}(m) = m.\frac{Fs}{N}$$

- Lembretes importantes:
  - A) “cada saída m da DFT é a soma termo a termo do produto de uma sequência de entrada no domínio n com uma sequência representando ondas seno e cosseno”;
  - B) para entradas de números reais, uma DFT de N pontos provê uma saída com N/2+1 termos independentes;
  - C) a magnitude dos resultados da DFT são proporcionais a N;
  - D) a resolução de frequência da DFT é dada pela forma acima

6) DFT inversa

- Fórmula:

$$x(n) = \frac{1}{N}\sum_{m=0}^{N-1} X(m)e^{j2\pi nm/N}$$

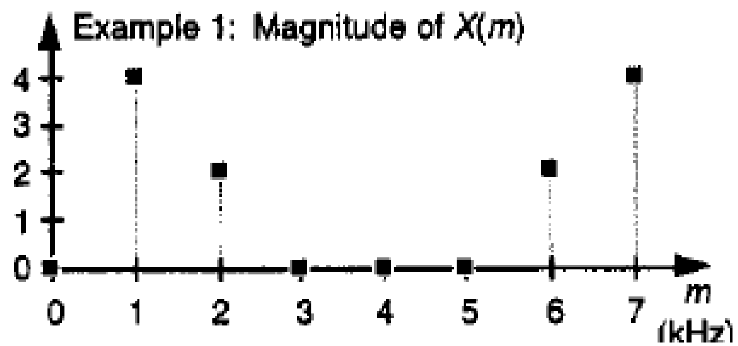
  – Aplicando ao exemplo 2 temos:

$x(0) = 0.3535 + j0.0$	$x(1) = 0.3535 + j0.0$
$x(2) = 0.6464 + j0.0$	$x(3) = 1.0607 + j0.0$
$x(4) = 0.3535 + j0.0$	$x(3) = -1.0607 + j0.0$
$x(6) = -1.3535 + j0.0$	$x(7) = -0.3535 + j0.0$

7) Leakage\vazamento

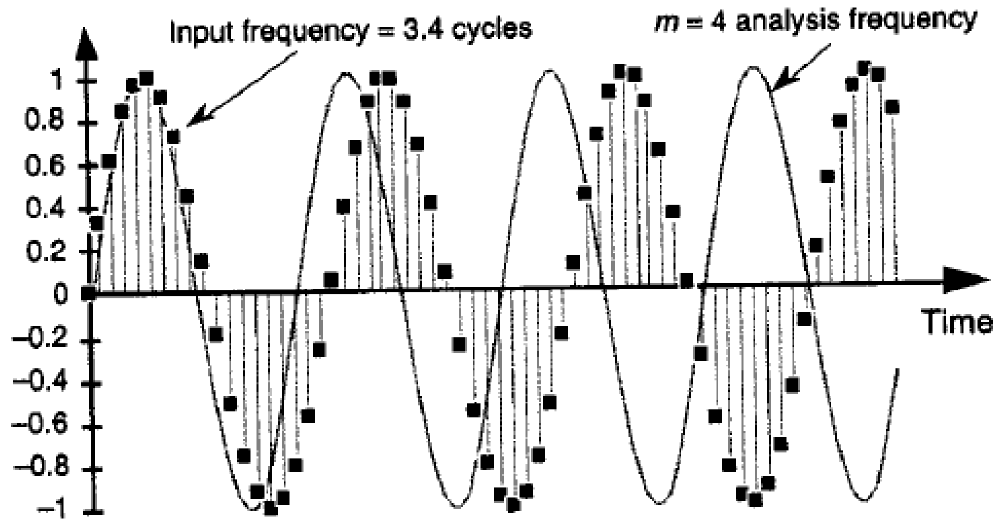
- Descrição problema:
  - Exemplo:

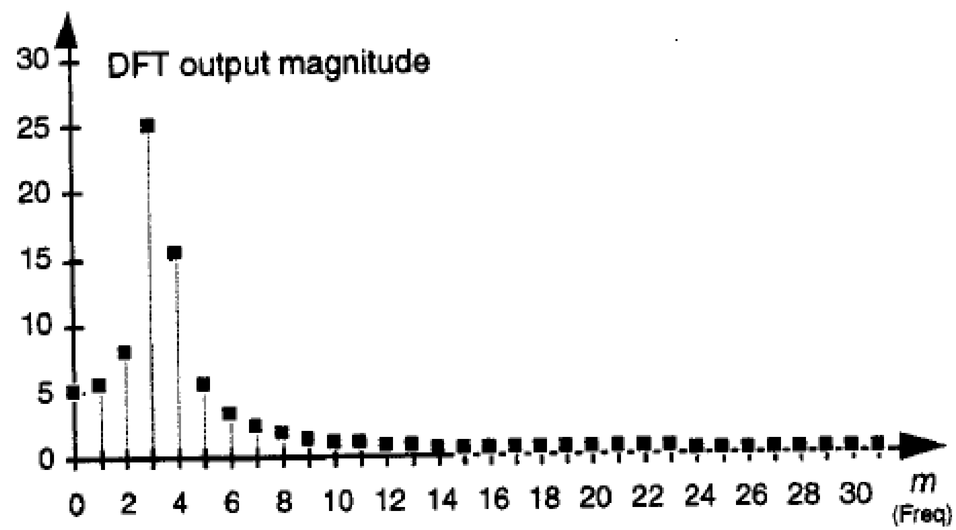
$$x(t) = sen(2\pi 1000t) + 0.5sen(2\pi 2000t + 3\pi/4)$$



Considerando Fs=8000 e N=8. E se Fs=5000?

– O leakage acontece quando a frequência de um sinal de entrada não tem correspondência exata com o eixo de frequência

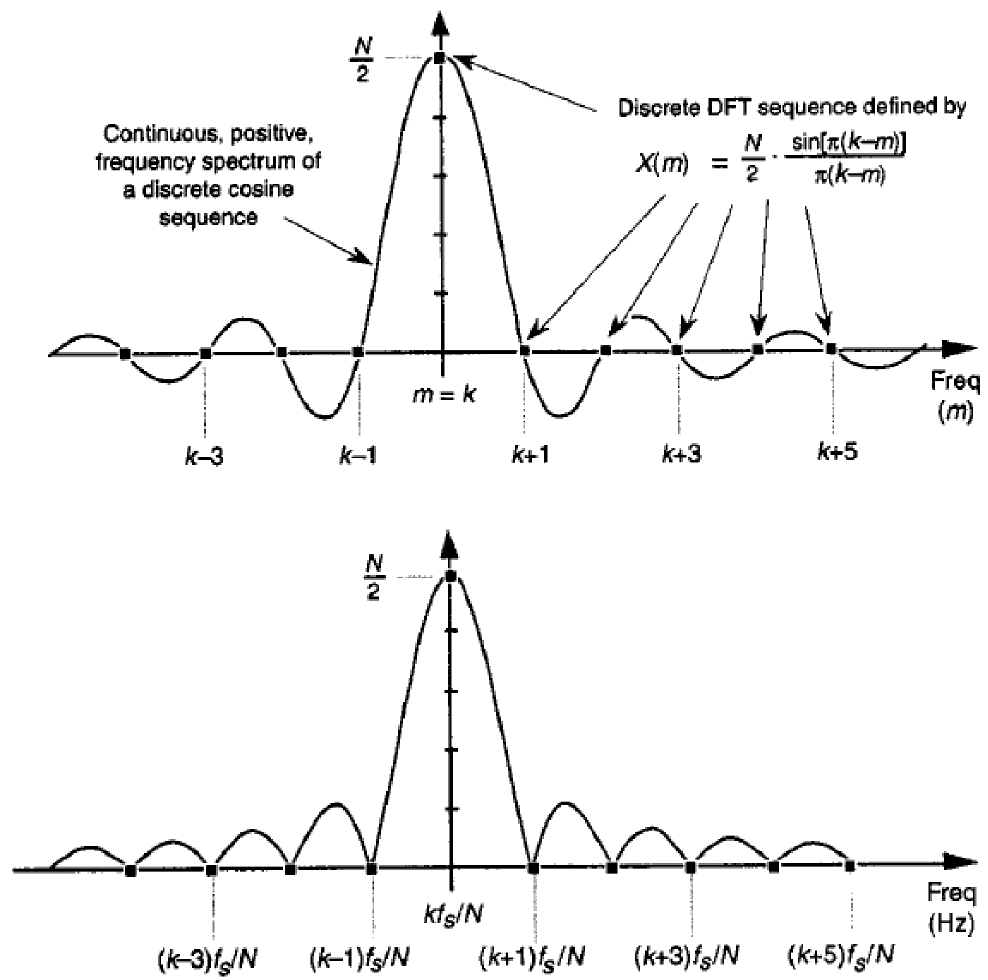




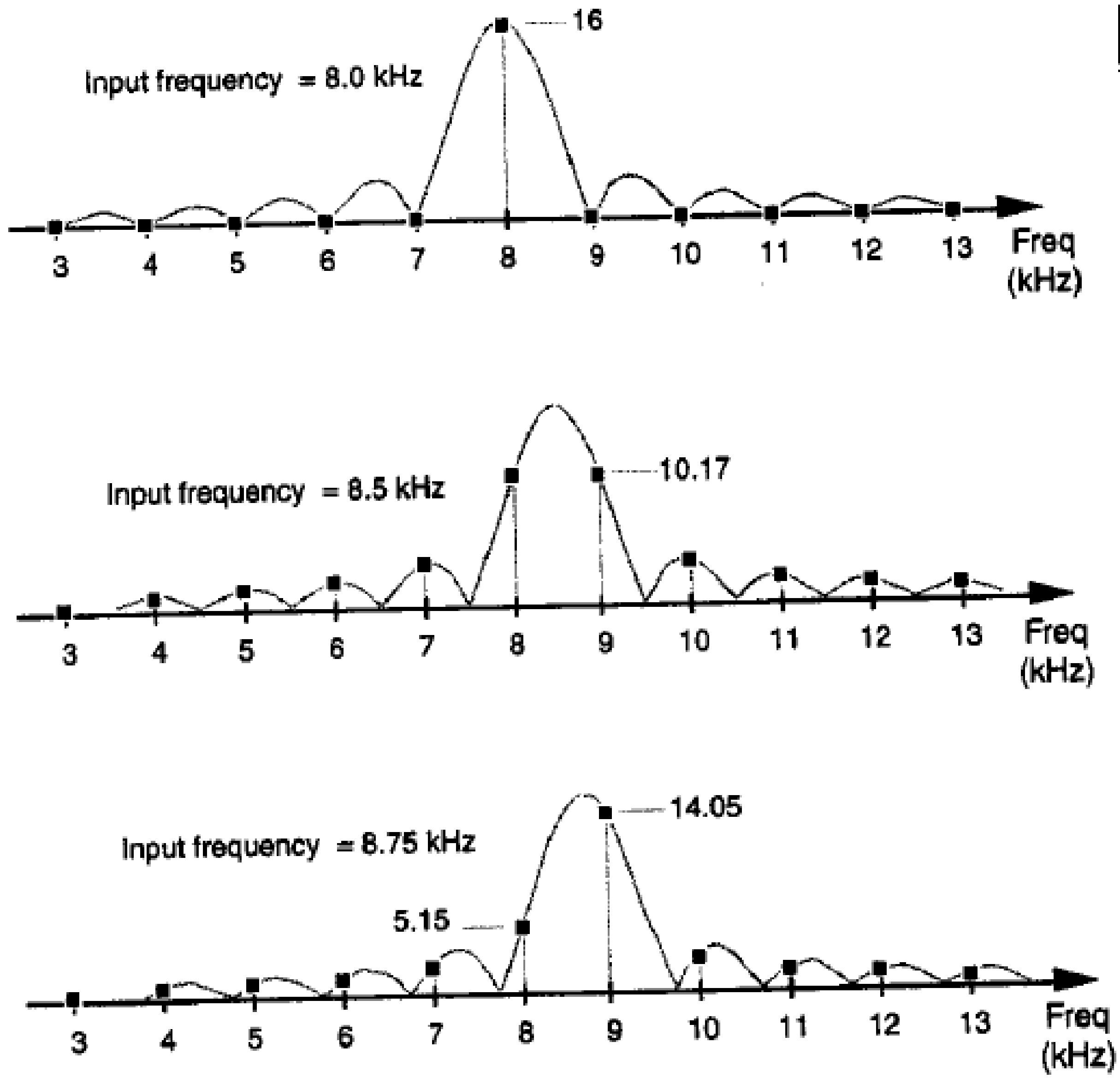
- Formato de um espectro puro:

$$X(m) = \frac{A_0 N}{2} \cdot \frac{\sin[\pi(k-m)]}{\pi(k-m)}$$

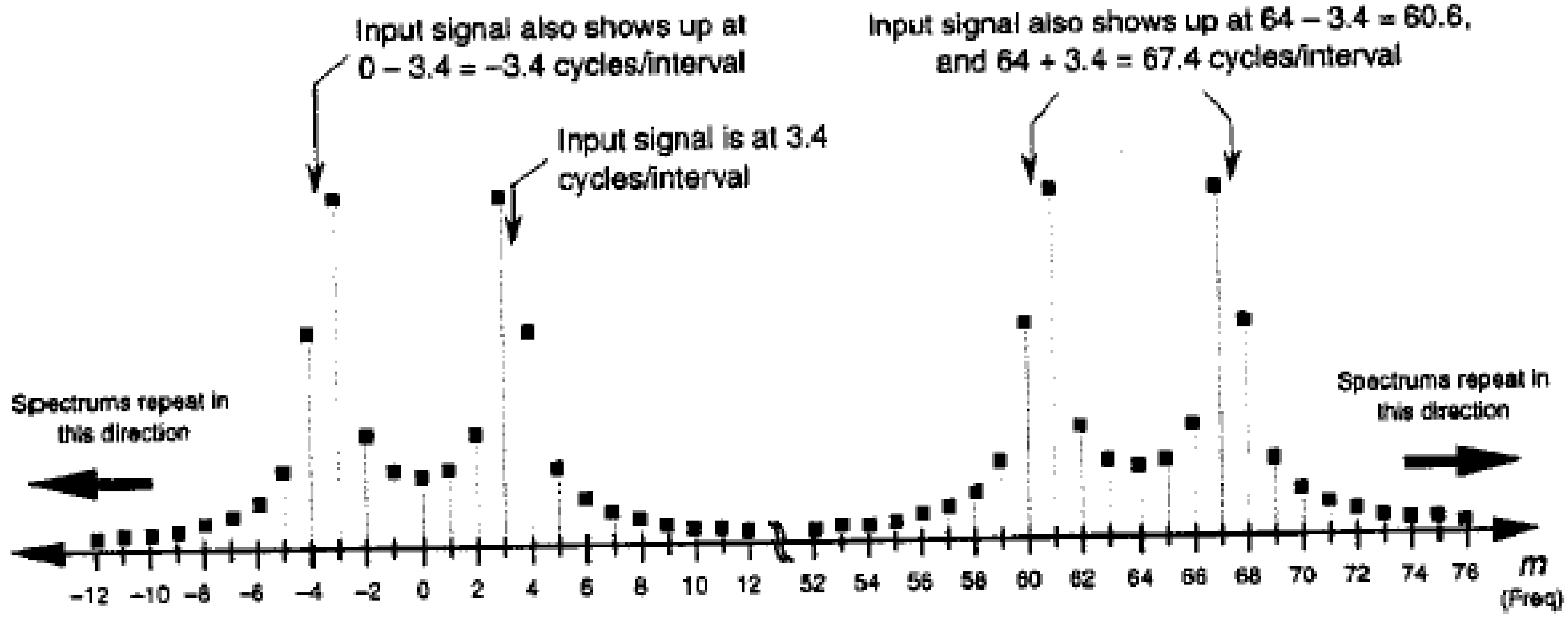
K=núm amostras por ciclo ou
Núm de ciclos da amostra



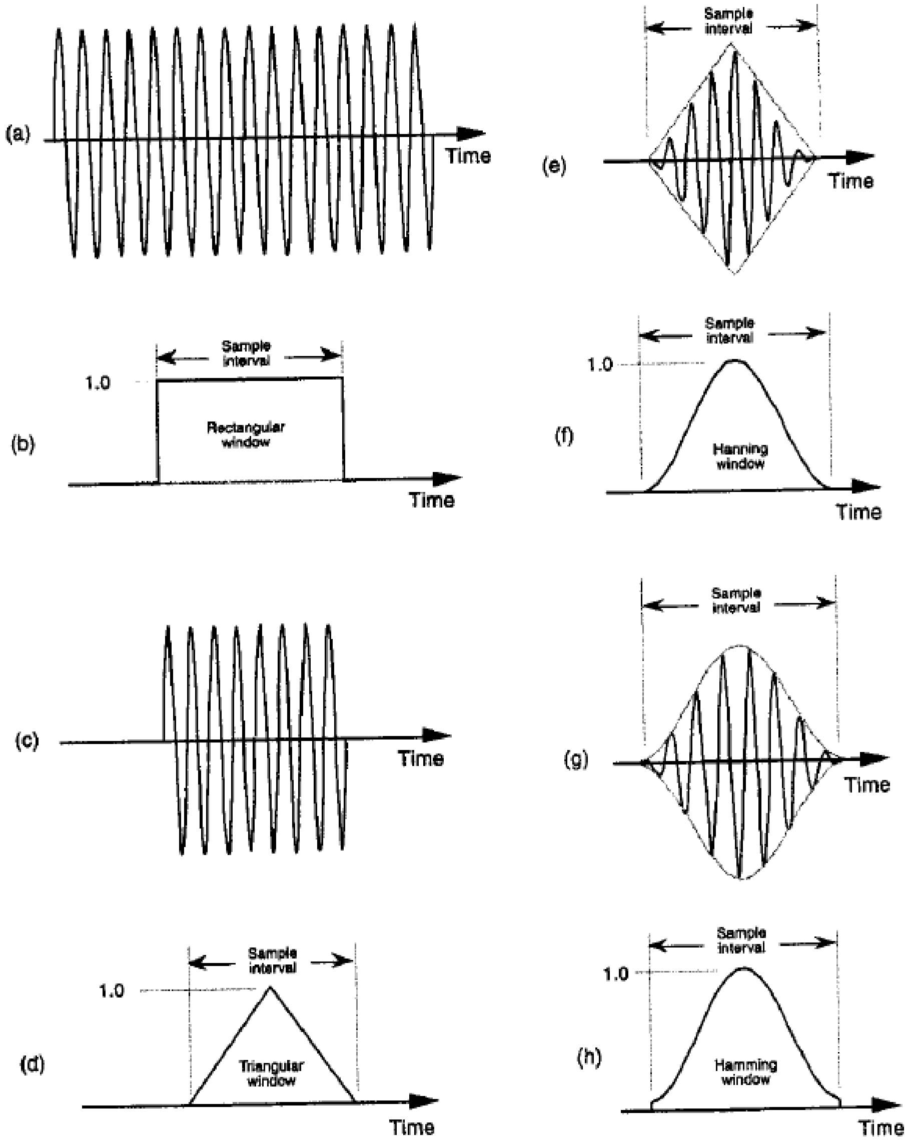
- Exemplo: Fs=32.000 e N=32



- Replicação espectral:



8) Janelamento



$$X_w(m) = \sum_{n=0}^{N-1} w(n)x(n)e^{-j2\pi nm/N}$$

Rectangular window:

$w(n) = 1$, for $n = 0, 1, 2, \ldots, N-1$.

Triangular window:

$$\omega(n) = \frac{n}{N/2}, \text{for } n = 0,1,2,\ldots,N/2,$$

$$= 2 - \frac{n}{N/2},$$

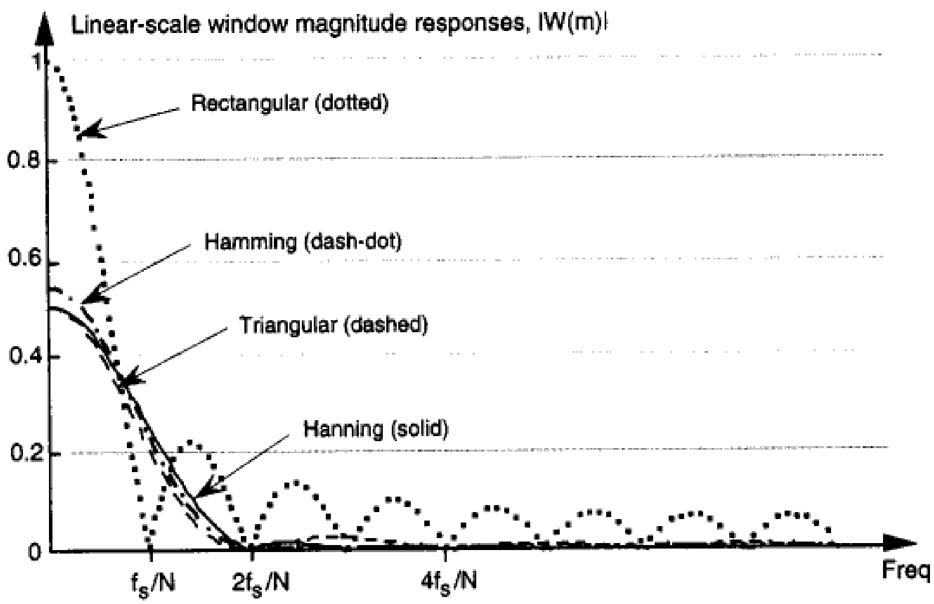
for $n = N/2+1, N/2+2, \ldots, N-1$.

Hanning window:

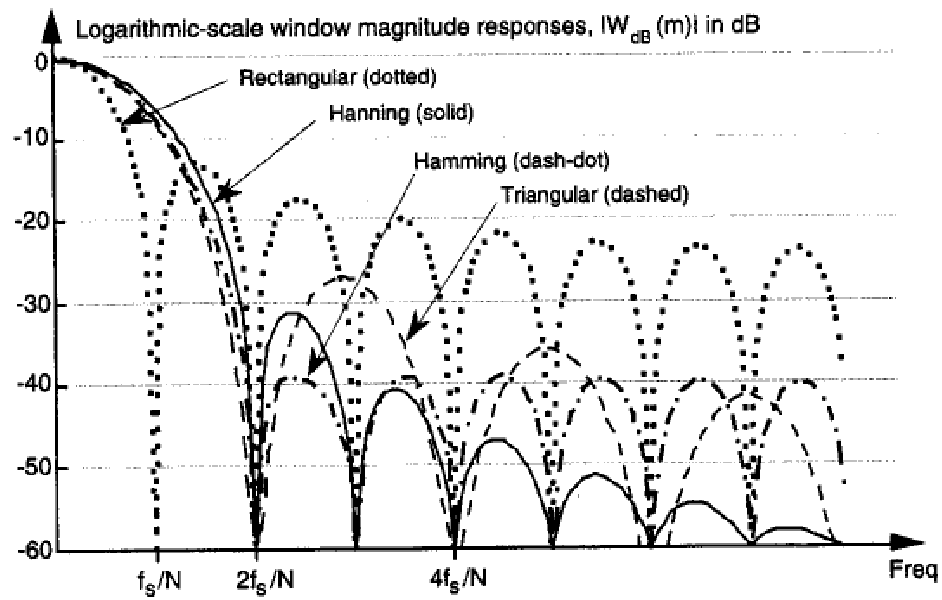
$w(n) = 0.5 - 0.5\cos(2\pi n/N\text{-}1)$,
for $n = 0, 1, 2, \ldots, N-1$.

Hamming window:

$w(n) = 0.54 - 0.46\cos(2\pi n/N\text{-}1)$,
for $n = 0, 1, 2, \ldots, N-1$.

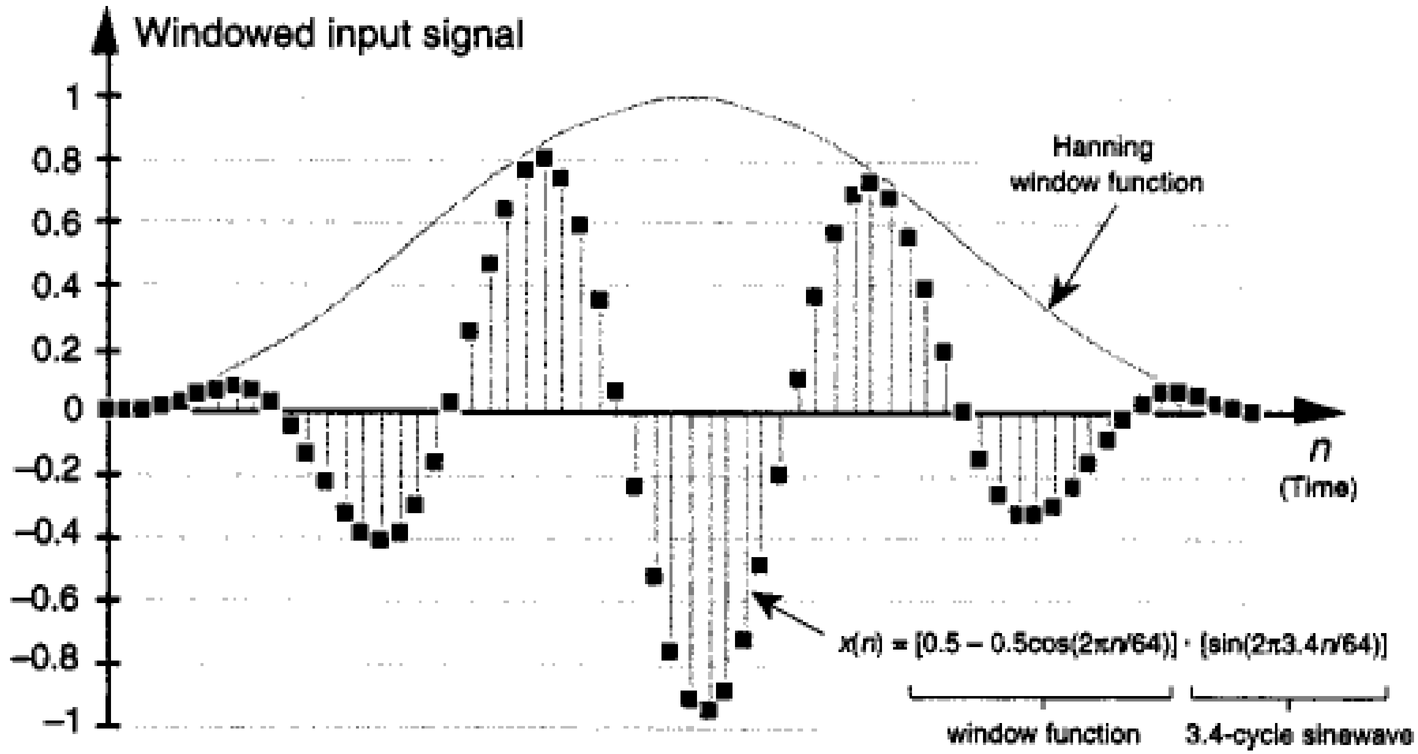


Linear-scale window magnitude responses, |W(m)|
1
0.8
0.6
0.4
0.2
0
Rectangular (dotted)
Hamming (dash-dot)
Triangular (dashed)
Hanning (solid)
fs/N
2fs/N
4fs/N
Freq

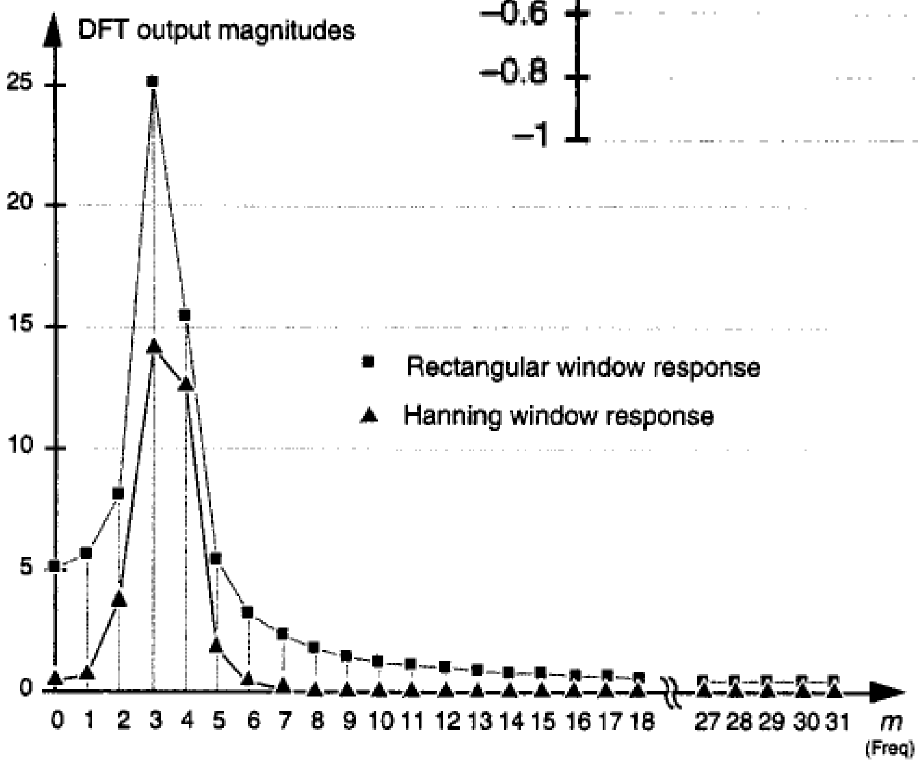


Logarithmic-scale window magnitude responses, |WdB (m)| in dB
0
-10
-20
-30
-40
-50
-60
Rectangular (dotted)
Hanning (solid)
Hamming (dash-dot)
Triangular (dashed)
fs/N
2fs/N
4fs/N
Freq

• Exemplo 3:

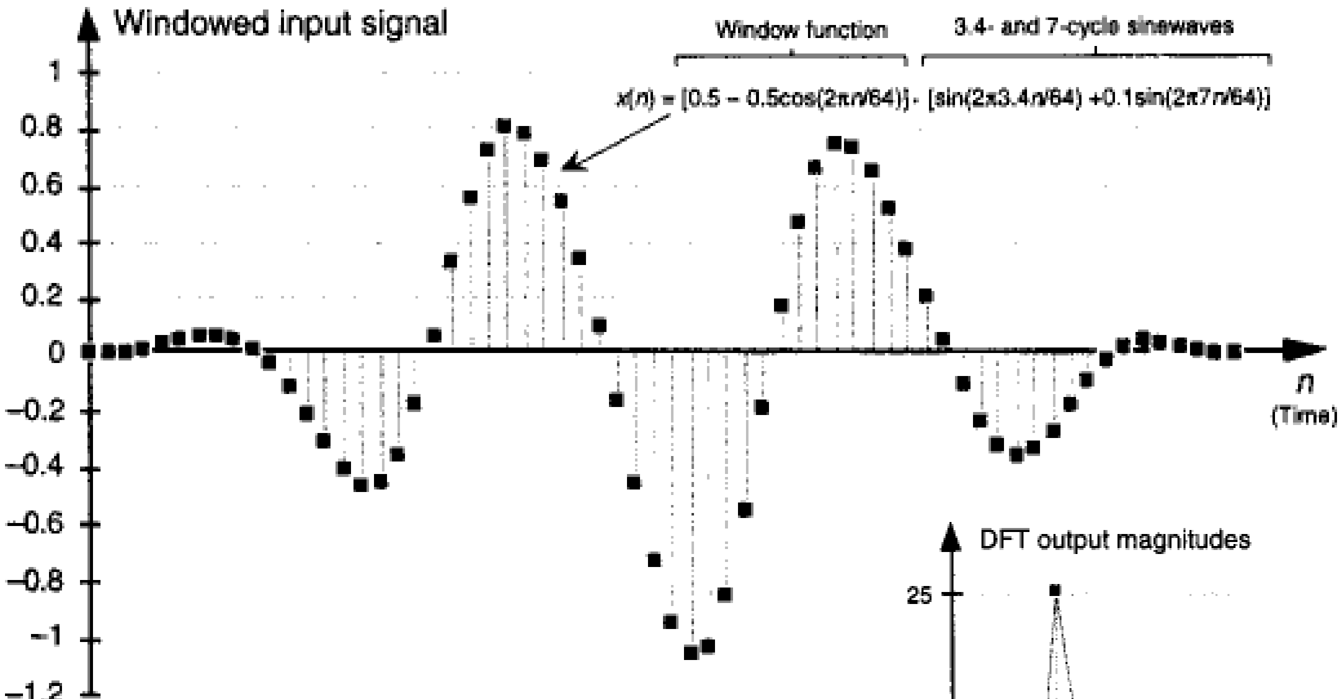


Windowed input signal
1
0.8
0.6
0.4
0.2
0
–0.2
–0.4
–0.6
–0.8
–1
Hanning window function
n (Time)
x(n) = [0.5 – 0.5cos(2πn/64)] · [sin(2π3.4n/64)]
window function
3.4-cycle sinewave

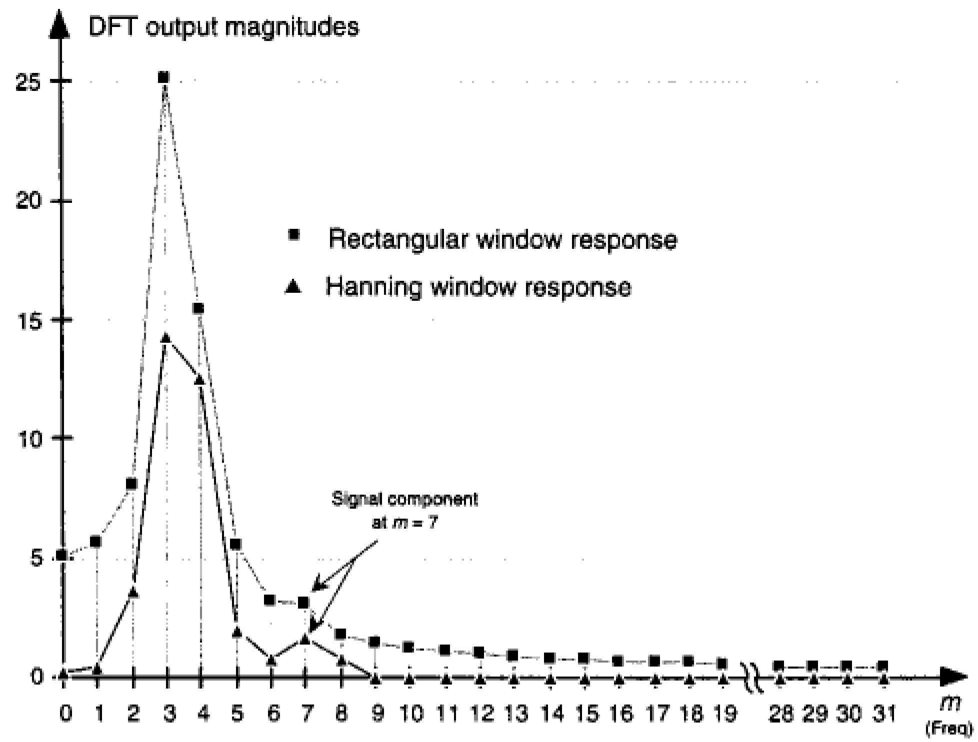


DFT output magnitudes
25
20
15
10
5
0
Rectangular window response
Hanning window response
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 27 28 29 30 31
m (Freq)

- Exemplo 4:



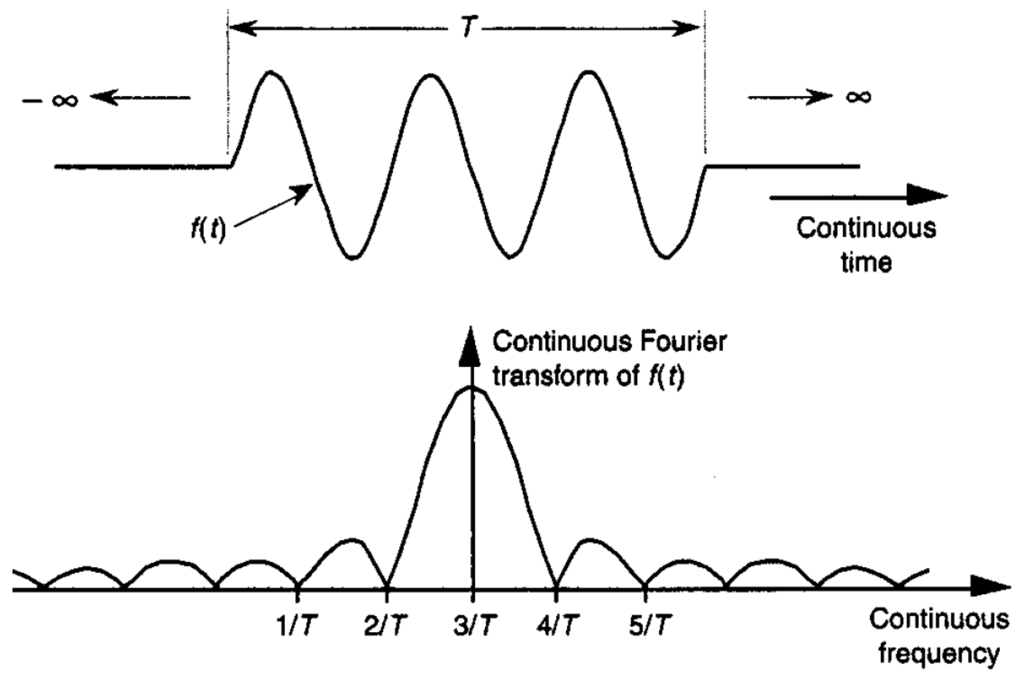
Windowed input signal
Window function
3.4- and 7-cycle sinewaves
$x(n) = [0.5 - 0.5\cos(2\pi n/64)] \cdot [\sin(2\pi 3.4n/64) + 0.1\sin(2\pi 7n/64)]$
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
n
(Time)



DFT output magnitudes
25
20
15
10
5
0
Rectangular window response
Hanning window response
Signal component at $m = 7$
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 28 29 30 31
m
(Freq)

9) Resolução e preenchimento com zeros

- Transformada contínua de Fourier:

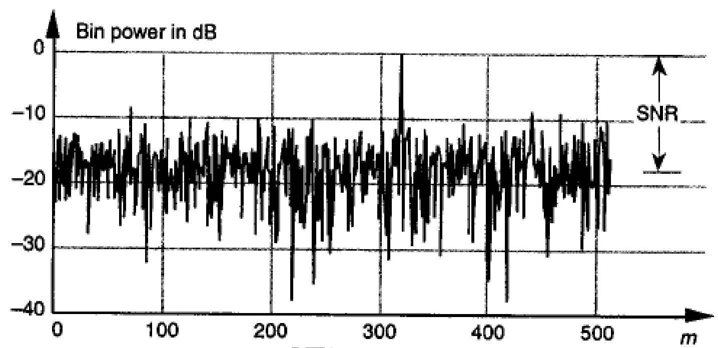


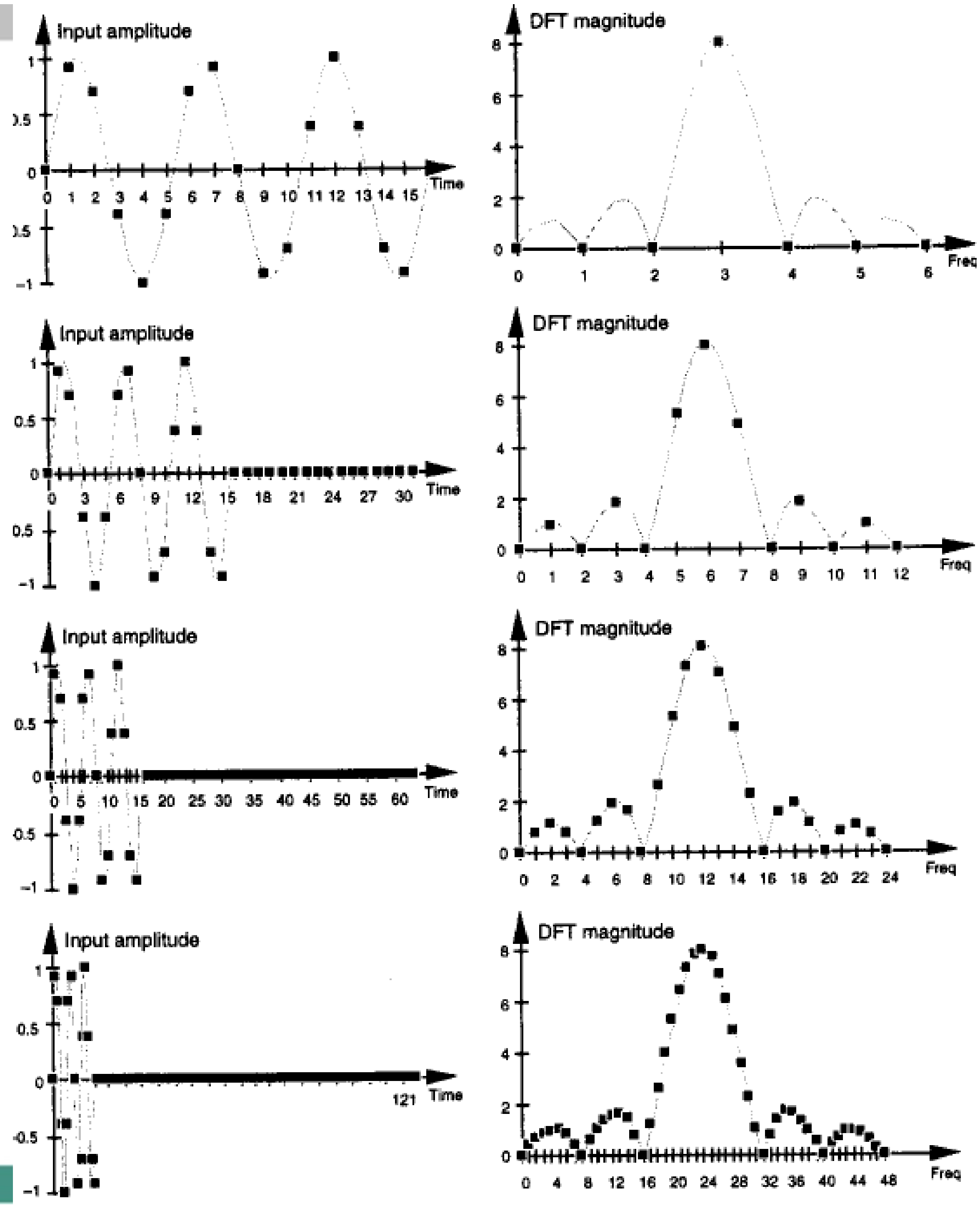
- O que fazer para melhorar a resolução do espectro?
  – Preencher a entrada com zeros?! (*zero padding*)

– Como “consertar” o eixo das frequências com preenchimento com zeros?

– Melhoramento da resolução mas não melhora entendimento das frequência de entrada

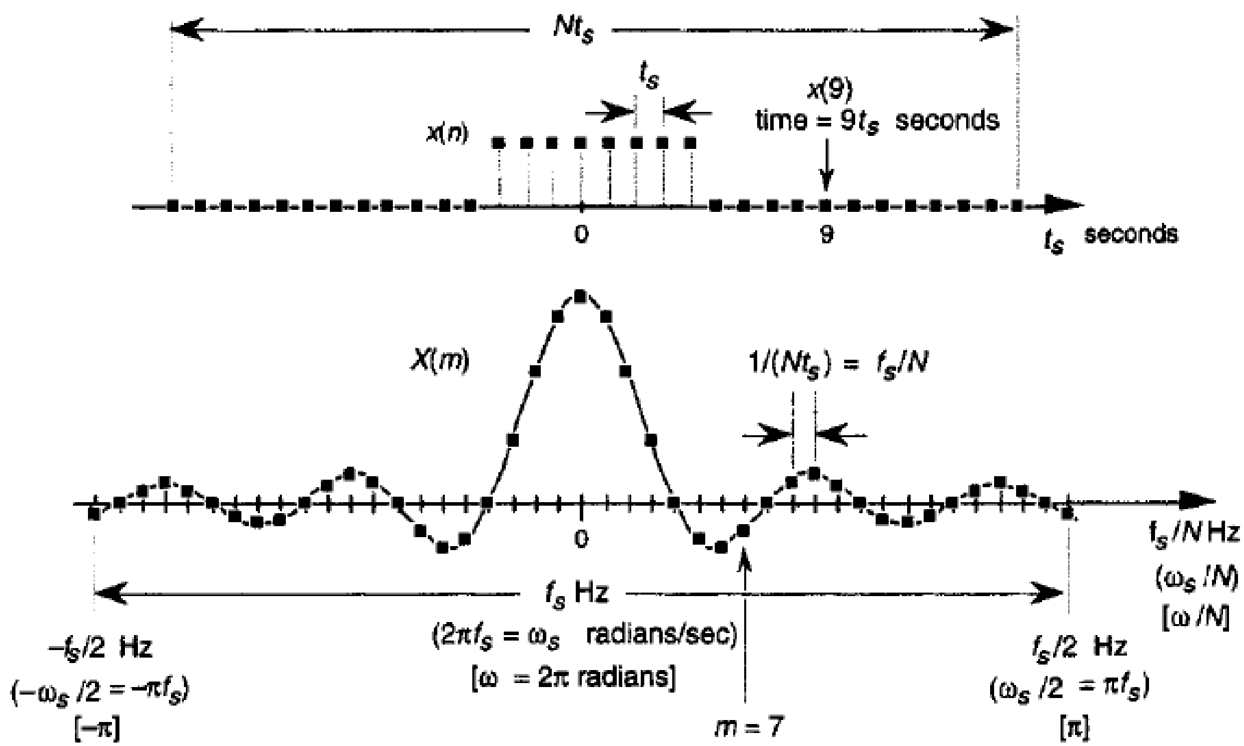
– Obs.: SNR





10) Representação no domínio da frequência

- Eixo da frequência em:
  - Hz ($f = 7.f_s/N$ Hz);
  - Normalizado por f_s ($f/f_s = 7/N$ ciclos/amostra);
  - Ângulo normalizado ($w=2\pi(f/f_s)$ radianos/amostra);



- Resumindo:

DFT Frequency Axis Representation	X(m) *Frequency Variable*	*Resolution of* X(m)	*Repetition Interval of* X(m)	*Frequency Axis Range*
Frequency in Hz	mf_s/N	f_s/N	f_s	$-f_s/2$ to $f_s/2$
Frequency in radians	$m\omega_s/N$ or $2\pi mf_s/N$	ω_s/N or $2\pi f_s/N$	ω_s or $2\pi f_s$	$-\omega_s/2$ to $\omega_s/2$ or $-\pi f_s$ to πf_s
Normalized angle in radians	$2\pi m/N$	$2\pi/N$	2π	$-\pi$ to π

- Representação alternativa:

$$X(w) = \sum_{n=-\infty}^{\infty} x(n) e^{-jwn}$$

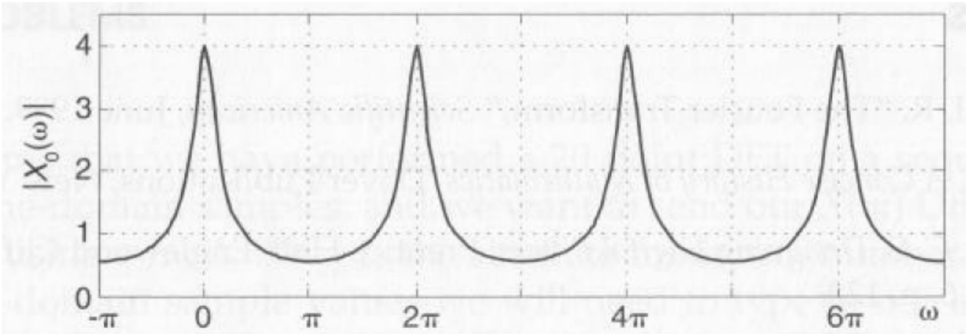
- Exemplo: calcule a transformada de `x(n)=(0,75)`^{n} `u(n)` usando a variável w.

Resolução:

$$X(w) = \sum_{n=-\infty}^{\infty} x(n)e^{-jwn}$$

$$X(w) = \sum_{n=0}^{\infty} 0{,}75^n e^{-jwn} = \sum_{n=0}^{\infty} (0{,}75e^{-jw})^n$$

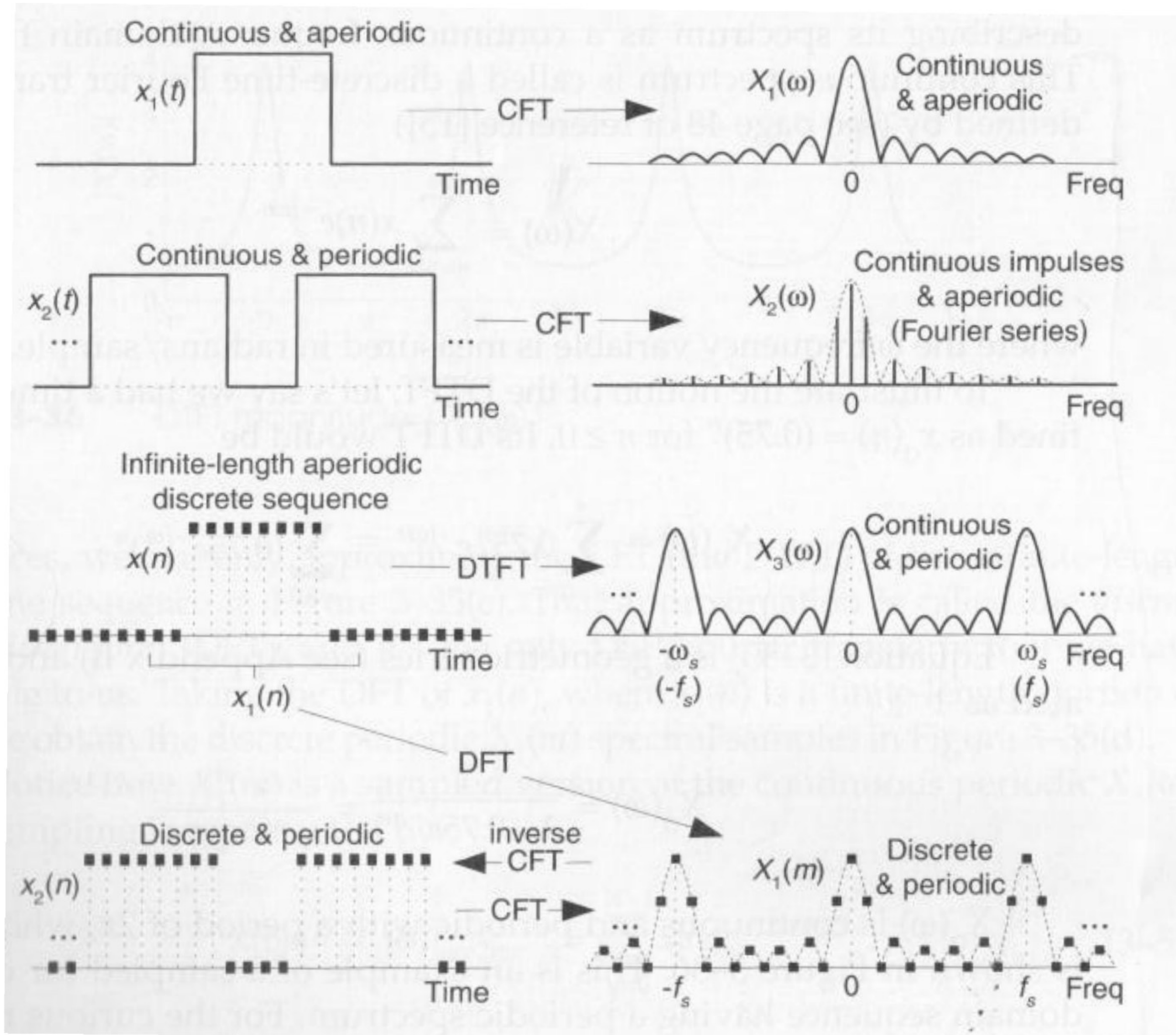
Aplicando a série geométrica: $X(w) = \dfrac{1}{1 - 0{,}75e^{-jw}}$



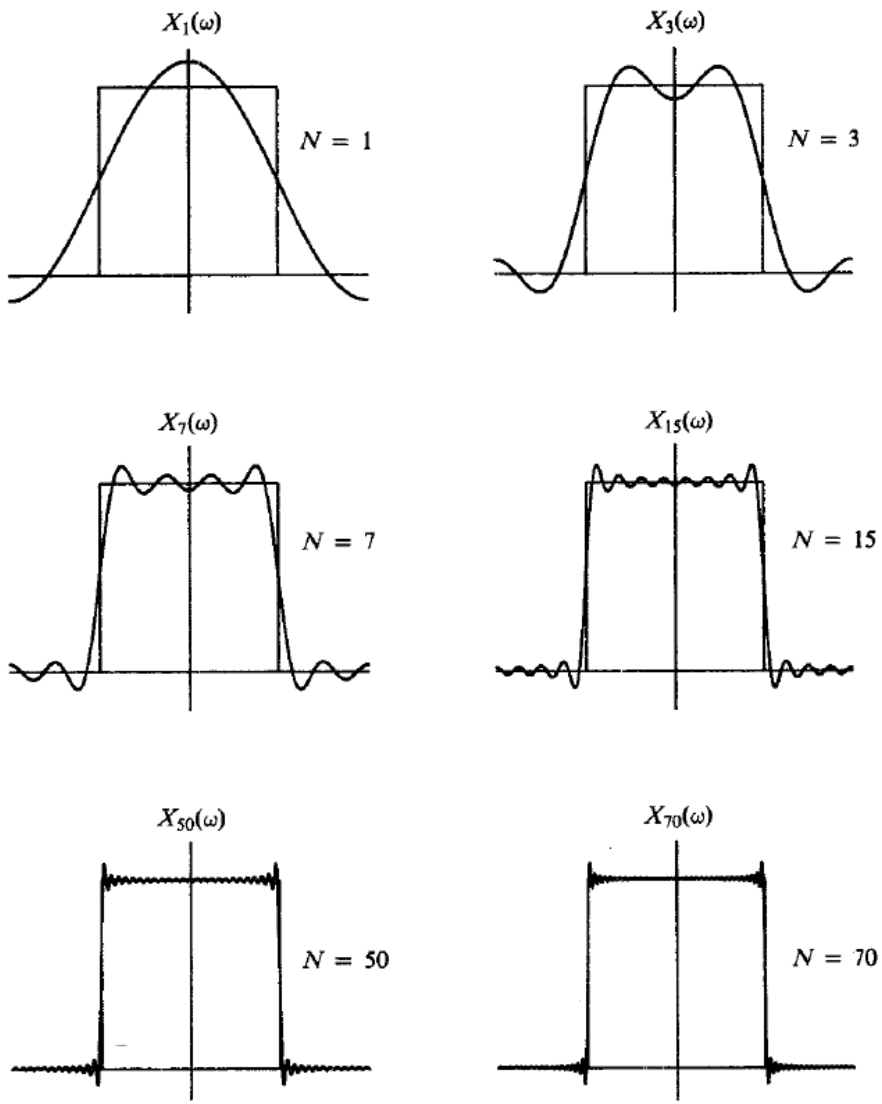
Conclusão:

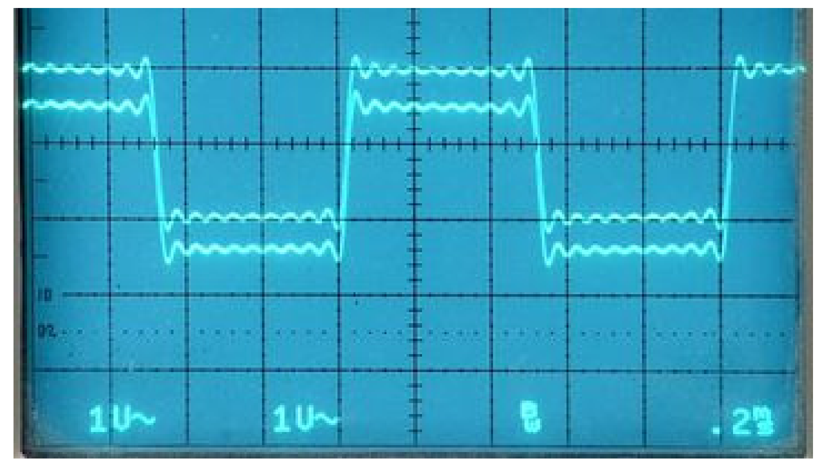
$$X(m) = X(w)\Big|_{w=2\pi m/N} = \sum_{n=0}^{N-1} x(n)e^{-j(2\pi m/N)n}$$

- Resumindo a: transformada contínua, a transformada discreta e a série de Fourier:



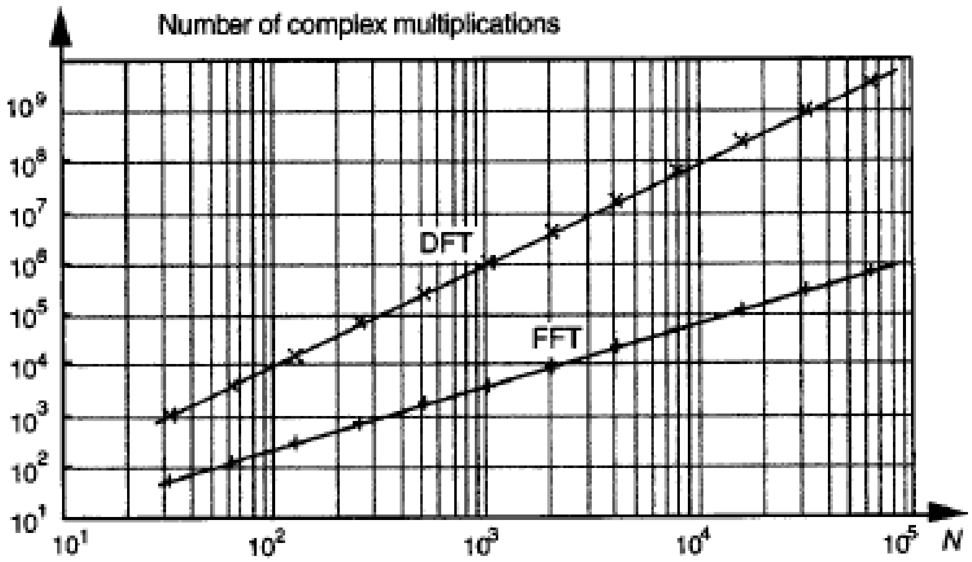
- Obs.: efeito de Gibbs em sistemas digitais





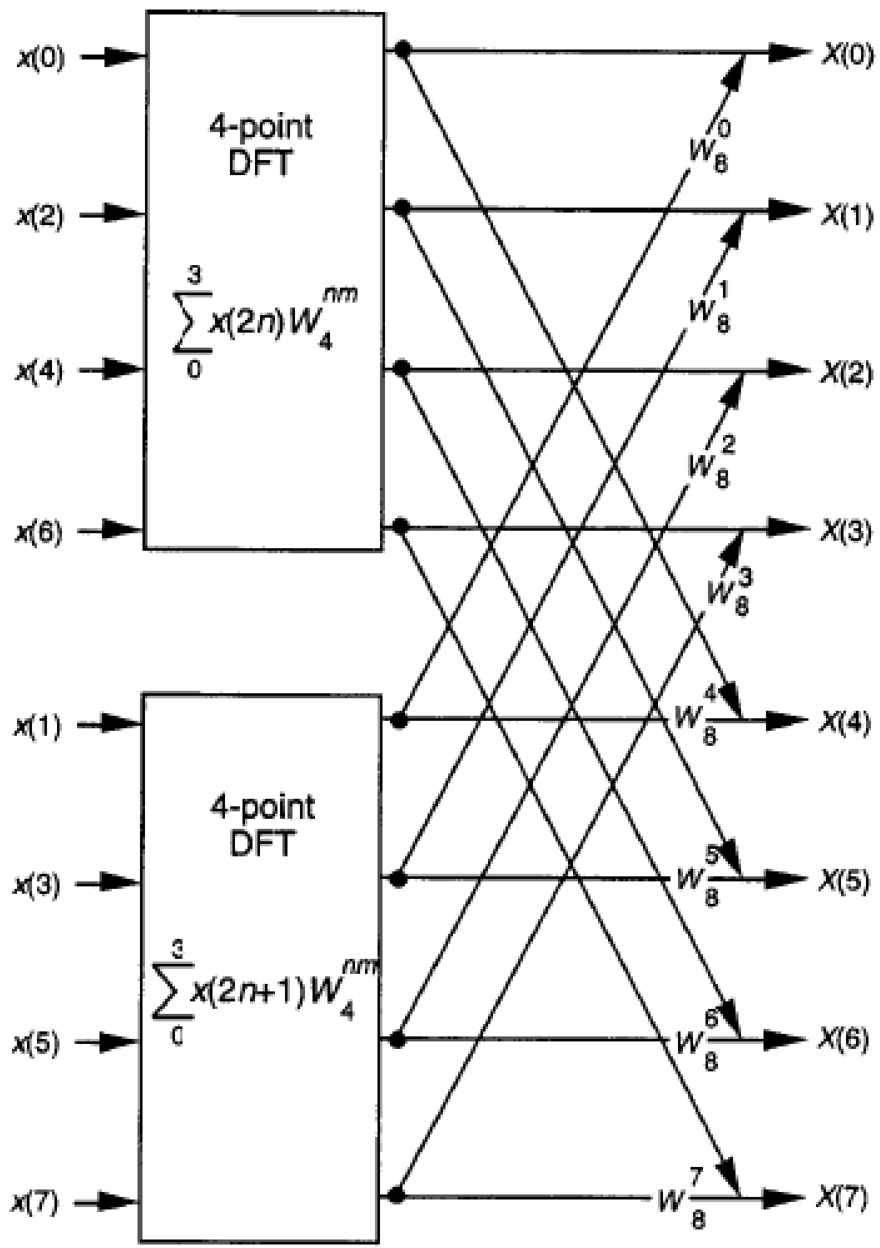
11) FFT

- Custo computacional



$$N^2 \quad \text{versus} \quad \frac{N}{2}.\log_2 N$$

- Para N = 2.097.152:
  - DFT: 3 semanas; FFT: 10 segundos!
- Potência de 2 (preenchimento com zeros)



x(0)
x(2)
x(4)
x(6)
4-point DFT
$\sum_{0}^{3} x(2n) W_4^{nm}$
x(1)
x(3)
x(5)
x(7)
4-point DFT
$\sum_{0}^{3} x(2n+1) W_4^{nm}$
W_8^0
W_8^1
W_8^2
W_8^3
W_8^4
W_8^5
W_8^6
W_8^7
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)