

FÓRMULAS DE DERIVADAS

- 1 $D_x c = 0$
- 2 $D_x (u + v) = D_x u + D_x v$
- 3 $D_x (uv) = u D_x v + v D_x u$
- 4 $D_x \left(\frac{u}{v} \right) = \frac{v D_x u - u D_x v}{v^2}$
- 5 $D_x f(g(x)) = f'(g(x)) g'(x)$
- 6 $D_x u^n = n u^{n-1} D_x u$
- 7 $D_x e^u = e^u D_x u$
- 8 $D_x a^u = a^u \ln a D_x u$
- 9 $D_x \ln |u| = \frac{1}{u} D_x u$
- 10 $D_x \log_a |u| = \frac{1}{u \ln a} D_x u$
- 11 $D_x \sin u = \cos u D_x u$
- 12 $D_x \cos u = -\sin u D_x u$
- 13 $D_x \operatorname{tg} u = \sec^2 u D_x u$
- 14 $D_x \cot u = -\operatorname{csc}^2 u D_x u$
- 15 $D_x \sec u = \sec u \operatorname{tg} u D_x u$
- 16 $D_x \csc u = -\csc u \cot u D_x u$
- 17 $D_x \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} D_x u$
- 18 $D_x \cos^{-1} u = \frac{-1}{\sqrt{1-u^2}} D_x u$
- 19 $D_x \operatorname{tg}^{-1} u = \frac{1}{1+u^2} D_x u$
- 20 $D_x \sec^{-1} u = \frac{1}{u \sqrt{u^2-1}} D_x u$

FÓRMULAS DE INTEGRAIS

- 1 $\int u dv = uv - \int v du$
- 2 $\int u^n du = \frac{1}{n+1} u^{n+1} + C, \quad n \neq -1$
- 3 $\int \frac{1}{u} du = \ln |u| + C$
- 4 $\int e^u du = e^u + C$
- 5 $\int a^u du = \frac{1}{\ln a} a^u + C$
- 6 $\int \sin u du = -\cos u + C$
- 7 $\int \cos u du = \sin u + C$
- 8 $\int \sec^2 u du = \operatorname{tg} u + C$
- 9 $\int \csc^2 u du = -\cot u + C$
- 10 $\int \sec u \operatorname{tg} u du = \sec u + C$
- 11 $\int \csc u \cot u du = -\csc u + C$
- 12 $\int \operatorname{tg} u du = -\ln |\cos u| + C$
- 13 $\int \cot u du = \ln |\sin u| + C$
- 14 $\int \sec u du = \ln |\sec u + \operatorname{tg} u| + C$
- 15 $\int \csc u du = \ln |\csc u - \cot u| + C$
- 16 $\int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1} \frac{u}{a} + C$
- 17 $\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \operatorname{tg}^{-1} \frac{u}{a} + C$
- 18 $\int \frac{1}{u \sqrt{u^2 - a^2}} du = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$
- 19 $\int \frac{1}{a^2 - u^2} du = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$
- 20 $\int \frac{1}{\sqrt{u^2 - a^2}} du = \ln |u + \sqrt{u^2 - a^2}| + C$

INTEGRAIS

$$1 \quad \int u \, dv = uv - \int v \, du$$

$$2 \quad \int u^n \, du = \frac{1}{n+1} u^{n+1} + C, \quad n \neq -1$$

$$3 \quad \int \frac{du}{u} = \ln |u| + C$$

$$4 \quad \int e^u \, du = e^u + C$$

$$5 \quad \int a^u \, du = \frac{1}{\ln a} a^u + C$$

$$6 \quad \int \operatorname{sen} u \, du = -\cos u + C$$

$$7 \quad \int \cos u \, du = \operatorname{sen} u + C$$

$$8 \quad \int \sec^2 u \, du = \operatorname{tg} u + C$$

$$9 \quad \int \csc^2 u \, du = -\cot u + C$$

$$10 \quad \int \sec u \operatorname{tg} u \, du = \sec u + C$$

$$11 \quad \int \csc u \cot u \, du = -\csc u + C$$

$$12 \quad \int \operatorname{tg} u \, du = -\ln |\cos u| + C$$

$$13 \quad \int \cot u \, du = \ln |\operatorname{sen} u| + C$$

$$14 \quad \int \sec u \, du = \ln |\sec u + \operatorname{tg} u| + C$$

$$15 \quad \int \csc u \, du = \ln |\csc u - \cot u| + C$$

$$16 \quad \int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{arcsen} \frac{u}{a} + C$$

$$17 \quad \int \frac{du}{a^2 + u^2} = \frac{1}{a} \operatorname{arctg} \frac{u}{a} + C$$

$$18 \quad \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{u}{a} + C$$

$$19 \quad \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$$

$$20 \quad \int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$$

Formas que envolvem $\sqrt{a^2 + u^2}$

$$21 \quad \int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln |u + \sqrt{a^2 + u^2}| + C$$

$$22 \quad \int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln |u + \sqrt{a^2 + u^2}| + C$$

$$23 \quad \int \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$$

$$24 \quad \int \frac{\sqrt{a^2 + u^2}}{u^2} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln |u + \sqrt{a^2 + u^2}| + C$$

$$25 \quad \int \frac{du}{\sqrt{a^2 + u^2}} = \ln |u + \sqrt{a^2 + u^2}| + C$$

$$26 \quad \int \frac{u^2 \, du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln |u + \sqrt{a^2 + u^2}| + C$$

$$27 \quad \int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$$

$$28 \quad \int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$$

$$29 \quad \int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$$

Formas que envolvem $\sqrt{a^2 - u^2}$

$$30 \quad \int \sqrt{a^2 - u^2} \, du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \arcsen \frac{u}{a} + C$$

$$31 \quad \int u^2 \sqrt{a^2 - u^2} \, du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \arcsen \frac{u}{a} + C$$

$$32 \quad \int \frac{\sqrt{a^2 - u^2}}{u} \, du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$33 \quad \int \frac{\sqrt{a^2 - u^2}}{u^2} \, du = -\frac{1}{u} \sqrt{a^2 - u^2} - \arcsen \frac{u}{a} + C$$

$$34 \quad \int \frac{u^2 \, du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \arcsen \frac{u}{a} + C$$

$$35 \quad \int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$36 \quad \int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$$

$$37 \quad \int (a^2 - u^2)^{3/2} \, du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \arcsen \frac{u}{a} + C$$

$$38 \quad \int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

Formas que envolvem $\sqrt{u^2 - a^2}$

$$39 \quad \int \sqrt{u^2 - a^2} \, du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$40 \quad \int u^2 \sqrt{u^2 - a^2} \, du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$41 \quad \int \frac{\sqrt{u^2 - a^2}}{u} \, du = \sqrt{u^2 - a^2} - a \arccos \frac{a}{u} + C$$

$$42 \quad \int \frac{\sqrt{u^2 - a^2}}{u^2} \, du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$$

$$43 \quad \int \frac{u^2 \, du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$44 \quad \int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

$$45 \quad \int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$$

$$46 \quad \int \frac{u^2 du}{(u^2 - a^2)^{3/2}} = \frac{-u}{\sqrt{u^2 - a^2}} + \ln |u + \sqrt{u^2 - a^2}| + C$$

Formas que envolvem $a + bu$

$$47 \quad \int \frac{u du}{a + bu} = \frac{1}{b^2} (a + bu - a \ln |a + bu|) + C$$

$$48 \quad \int \frac{u^2 du}{a + bu} = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|] + C$$

$$49 \quad \int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$$

$$50 \quad \int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$$

$$51 \quad \int \frac{u du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2} \ln |a + bu| + C$$

$$52 \quad \int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$$

$$53 \quad \int \frac{u^2 du}{(a + bu)^2} = \frac{1}{b^3} \left(a + bu - \frac{a^2}{a + bu} - 2a \ln |a + bu| \right) + C$$

$$54 \quad \int u \sqrt{a + bu} du = \frac{2}{15b^2} (3bu - 2a)(a + bu)^{3/2} + C$$

$$55 \quad \int \frac{u du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$$

$$56 \quad \int \frac{u^2 du}{\sqrt{a + bu}} = \frac{2}{15b^3} (8a^2 + 3b^2u^2 - 4abu) \sqrt{a + bu} + C$$

$$57 \quad \int \frac{du}{u\sqrt{a + bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, \text{ se } a > 0$$

$$= \frac{2}{\sqrt{-a}} \operatorname{arctg} \sqrt{\frac{a+1}{-a}} + C, \text{ se } a < 0$$

$$58 \quad \int \frac{\sqrt{a + bu}}{u} du = 2\sqrt{a + bu} + a \int \frac{du}{u\sqrt{a + bu}}$$

$$59 \quad \int \frac{\sqrt{a + bu}}{u^2} du = -\frac{\sqrt{a + bu}}{u} + \frac{b}{2} \int \frac{du}{u\sqrt{a + bu}}$$

$$60 \quad \int u^n \sqrt{a + bu} du = \frac{2}{b(2n+3)} \left[u^n(a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} du \right]$$

$$61 \quad \int \frac{u^n du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n+1)} - \frac{2na}{b(2n+1)} \int \frac{u^{n-1} du}{\sqrt{a + bu}}$$

$$62 \quad \int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)} \int \frac{du}{u^{n-1} \sqrt{a + bu}}$$

Formas trigonométricas

$$63 \quad \int \sen^2 u \, du = \frac{1}{2} u - \frac{1}{4} \sen 2u + C$$

$$64 \quad \int \cos^2 u \, du = \frac{1}{2} u + \frac{1}{4} \sen 2u + C$$

$$65 \quad \int \tg^2 u \, du = \tg u - u + C$$

$$66 \quad \int \cot^2 u \, du = -\cot u - u + C$$

$$67 \quad \int \sen^3 u \, du = -\frac{1}{3} (2 + \sen^2 u) \cos u + C$$

$$68 \quad \int \cos^3 u \, du = \frac{1}{3} (2 + \cos^2 u) \sen u + C$$

$$69 \quad \int \tg^3 u \, du = \frac{1}{2} \tg^2 u + \ln |\cos u| + C$$

$$70 \quad \int \cot^3 u \, du = -\frac{1}{2} \cot^2 u - \ln |\sen u| + C$$

$$71 \quad \int \sec^3 u \, du = \frac{1}{2} \sec u \tg u + \frac{1}{2} \ln |\sec u + \tg u| + C$$

$$72 \quad \int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$$

$$73 \quad \int \sen^n u \, du = -\frac{1}{n} \sen^{n-1} u \cos u + \frac{n-1}{n} \int \sen^{n-2} u \, du$$

$$74 \quad \int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sen u + \frac{n-1}{n} \int \cos^{n-2} u \, du$$

$$75 \quad \int \tg^n u \, du = \frac{1}{n-1} \tg^{n-1} u - \int \tg^{n-2} u \, du$$

$$76 \quad \int \cot^n u \, du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du$$

$$77 \quad \int \sec^n u \, du = \frac{1}{n-1} \tg u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$$

$$78 \quad \int \csc^n u \, du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$$

$$79 \quad \int \sen au \sen bu \, du = \frac{\sen (a-b)u}{2(a-b)} - \frac{\sen (a+b)u}{2(a+b)} + C$$

$$80 \quad \int \cos au \cos bu \, du = \frac{\sen (a-b)u}{2(a-b)} + \frac{\sen (a+b)u}{2(a+b)} + C$$

$$81 \quad \int \sen au \cos bu \, du = -\frac{\cos (a-b)u}{2(a-b)} - \frac{\cos (a+b)u}{2(a+b)} + C$$

$$82 \quad \int u \sen u \, du = \sen u - u \cos u + C$$

$$83 \quad \int u \cos u \, du = \cos u + u \sen u + C$$

$$84 \quad \int u^n \sen u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$$

$$85 \quad \int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$$

$$86 \quad \int \sin^n u \cos^m u \, du = -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u \, du$$

$$= \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \cos^{m-2} u \, du$$

Formas trigonométricas inversas

$$87 \quad \int \arcsen u \, du = u \arcsen u + \sqrt{1-u^2} + C$$

$$88 \quad \int \arccos u \, du = u \arccos u - \sqrt{1-u^2} + C$$

$$89 \quad \int \operatorname{arctg} u \, du = u \operatorname{arctg} u - \frac{1}{2} \ln(1+u^2) + C$$

$$90 \quad \int \arcsen u \, du = \frac{2u^2-1}{4} u \arcsen u + \frac{u\sqrt{1-u^2}}{4} + C$$

$$91 \quad \int u \arccos u \, du = \frac{2u^2-1}{4} \arccos u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$92 \quad \int u \operatorname{arctg} u \, du = \frac{u^2+1}{2} \operatorname{arctg} u - \frac{u}{2} + C$$

$$93 \quad \int u^n \arcsen u \, du = \frac{1}{n+1} \left[u^{n+1} \arcsen u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$$

$$94 \quad \int u^n \arccos u \, du = \frac{1}{n+1} \left[u^{n+1} \arccos u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$$

$$95 \quad \int u^n \operatorname{arctg} u \, du = \frac{1}{n+1} \left[u^{n+1} \operatorname{arctg} u - \int \frac{u^{n+1} du}{1+u^2} \right], \quad n \neq -1$$

Formas exponenciais e logarítmicas

$$96 \quad \int u e^{au} \, du = \frac{1}{a^2} (au-1)e^{au} + C$$

$$97 \quad \int u^n e^{au} \, du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} \, du$$

$$98 \quad \int e^{au} \sin bu \, du = \frac{e^{au}}{a^2+b^2} (a \sin bu - b \cos bu) + C$$

$$99 \quad \int e^{au} \cos bu \, du = \frac{e^{au}}{a^2+b^2} (a \cos bu + b \sin bu) + C$$

$$100 \quad \int \ln u \, du = u \ln u - u + C$$

$$101 \quad \int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$$

$$102 \quad \int \frac{1}{u \ln u} du = \ln |\ln u| + C$$

Formas hiperbólicas

$$103 \quad \int \sinh u \, du = \cosh u + C$$

$$104 \quad \int \cosh u \, du = \sinh u + C$$

$$105 \quad \int \operatorname{tgh} u \, du = \ln \cosh u + C$$

$$106 \quad \int \operatorname{coth} u \, du = \ln |\sinh u| + C$$

$$107 \quad \int \operatorname{sech} u \, du = \operatorname{arctgh} \sinh u + C$$

$$108 \quad \int \operatorname{csch} u \, du = \ln |\operatorname{tgh} \frac{1}{2} u| + C$$

$$109 \quad \int \operatorname{sech}^2 u \, du = \operatorname{tgh} u + C$$

$$110 \quad \int \operatorname{csch}^2 u \, du = -\operatorname{coth} u + C$$

$$111 \quad \int \operatorname{sech} u \operatorname{tgh} u \, du = -\operatorname{sech} u + C$$

$$112 \quad \int \operatorname{csch} u \operatorname{coth} u \, du = -\operatorname{csch} u + C$$

Formas que envolvem $\sqrt{2au - u^2}$

$$113 \quad \int \sqrt{2au - u^2} \, du = \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \arccos \left(\frac{a-u}{a} \right) + C$$

$$114 \quad \int u \sqrt{2au - u^2} \, du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \arccos \left(\frac{a-u}{a} \right) + C$$

$$115 \quad \int \frac{\sqrt{2au - u^2}}{u} \, du = \sqrt{2au - u^2} + a \arccos \left(\frac{a-u}{a} \right) + C$$

$$116 \quad \int \frac{\sqrt{2au - u^2}}{u^2} \, du = -\frac{2\sqrt{2au - u^2}}{u} \arccos \left(\frac{a-u}{a} \right) + C$$

$$117 \quad \int \frac{du}{\sqrt{2au - u^2}} = \arccos \left(\frac{a-u}{a} \right) + C$$

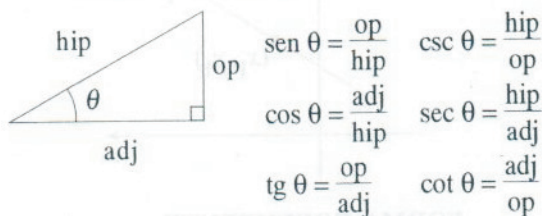
$$118 \quad \int \frac{u \, du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \arccos \left(\frac{a-u}{a} \right) + C$$

$$119 \quad \int \frac{u^2 \, du}{\sqrt{2au - u^2}} = -\frac{(u+3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \arccos \left(\frac{a-u}{a} \right) + C$$

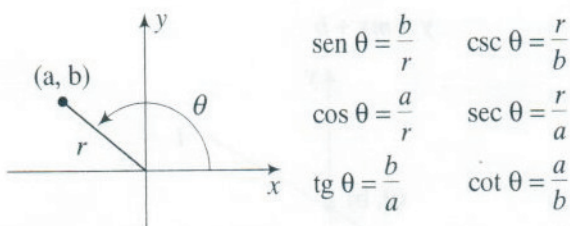
$$120 \quad \int \frac{du}{u \sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$$

RELAÇÕES TRIGONOMÉTRICAS

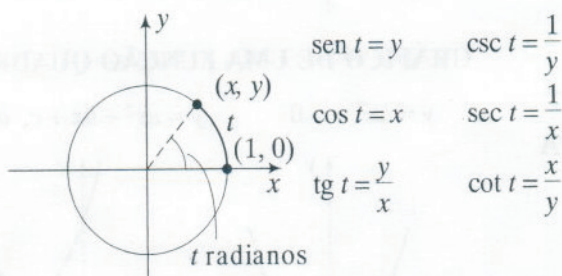
FUNÇÕES TRIGONOMÉTRICAS DE ÂNGULOS AGUDOS



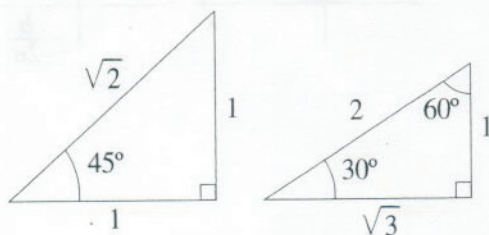
DE ÂNGULOS ARBITRÁRIOS



DE NÚMEROS REAIS



TRIÂNGULOS ESPECIAIS



IDENTIDADES TRIGONOMÉTRICAS

$$\begin{aligned} \text{csc } t &= \frac{1}{\text{sen } t} & \text{tg } t &= \frac{\text{sen } t}{\text{cos } t} \\ \text{sec } t &= \frac{1}{\text{cos } t} & \text{cot } t &= \frac{\text{cos } t}{\text{sen } t} \\ \text{cot } t &= \frac{1}{\text{tg } t} \\ \text{sen}^2 t + \text{cos}^2 t &= 1 & \text{sen}(-t) &= -\text{sen } t \\ 1 + \text{tg}^2 t &= \text{sec}^2 t & \text{cos}(-t) &= \text{cos } t \\ 1 + \text{cot}^2 t &= \text{csc}^2 t & \text{tg}(-t) &= -\text{tg } t \\ \text{sen}(u + v) &= \text{sen } u \text{ cos } v + \text{cos } u \text{ sen } v \end{aligned}$$

$$\text{cos}(u + v) = \text{cos } u \text{ cos } v - \text{sen } u \text{ sen } v$$

$$\text{tg}(u + v) = \frac{\text{tg } u + \text{tg } v}{1 - \text{tg } u \text{ tg } v}$$

$$\text{sen}(u - v) = \text{sen } u \text{ cos } v - \text{cos } u \text{ sen } v$$

$$\text{cos}(u - v) = \text{cos } u \text{ cos } v + \text{sen } u \text{ sen } v$$

$$\text{tg}(u - v) = \frac{\text{tg } u - \text{tg } v}{1 + \text{tg } u \text{ tg } v}$$

$$\text{sen } 2u = 2 \text{ sen } u \text{ cos } u$$

$$\text{cos } 2u = \text{cos}^2 u - \text{sen}^2 u = 1 - 2 \text{ sen}^2 u = 2 \text{ cos}^2 u - 1$$

$$\text{tg } 2u = \frac{2 \text{ tg } u}{1 - \text{tg}^2 u}$$

$$\left| \text{sen } \frac{u}{2} \right| = \sqrt{\frac{1 - \text{cos } u}{2}} \quad \left| \text{cos } \frac{u}{2} \right| = \sqrt{\frac{1 + \text{cos } u}{2}}$$

$$\text{tg } \frac{u}{2} = \frac{1 - \text{cos } u}{\text{sen } u} = \frac{\text{sen } u}{1 + \text{cos } u}$$

$$\text{sen}^2 u = \frac{1 - \text{cos } 2u}{2} \quad \text{cos}^2 u = \frac{1 + \text{cos } 2u}{2}$$

$$\text{sen } u \text{ cos } v = \frac{1}{2} [\text{sen}(u + v) + \text{sen}(u - v)]$$

$$\text{cos } u \text{ sen } v = \frac{1}{2} [\text{sen}(u + v) - \text{sen}(u - v)]$$

$$\text{cos } u \text{ cos } v = \frac{1}{2} [\text{cos}(u + v) + \text{cos}(u - v)]$$

$$\text{sen } u \text{ sen } v = \frac{1}{2} [\text{cos}(u - v) - \text{cos}(u + v)]$$

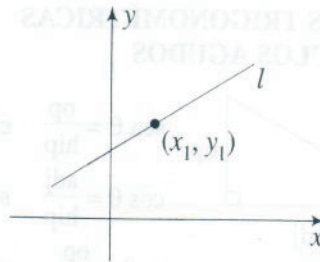
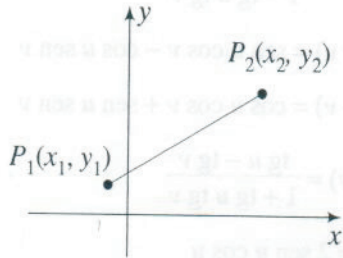
VALORES ESPECIAIS DE FUNÇÕES TRIGONOMÉTRICAS

θ (graus)	θ (radianos)	sen θ	cos θ	tg θ	cot θ	sec θ	csc θ
0°	0	0	1	0	—	1	—
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	—	0	—	1

GEOMETRIA ANALÍTICA

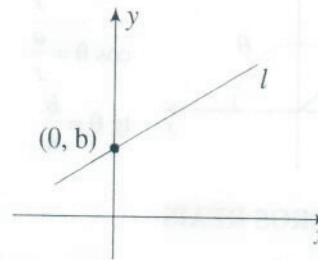
FÓRMULA DA DISTÂNCIA

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



FORMA COEFICIENTE ANGULAR-INTERCEPTO

$$y = mx + b$$



EQUAÇÃO DE UM CÍRCULO

$$(x - h)^2 + (y - k)^2 = r^2$$

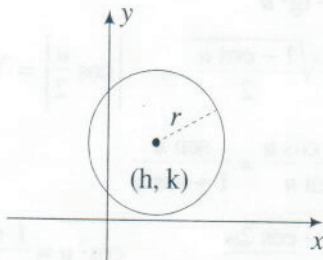
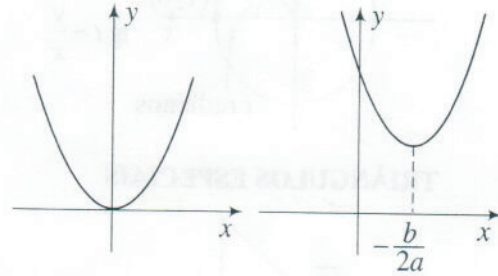


GRÁFICO DE UMA FUNÇÃO QUADRÁTICA

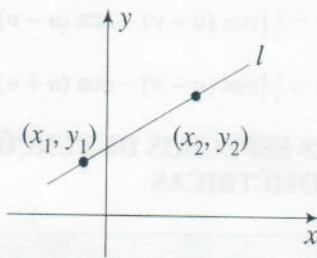
$$y = ax^2, a > 0$$

$$y = ax^2 + bx + c, a > 0$$



COEFICIENTE ANGULAR m DE UMA RETA

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



FORMA PONTO-COEFICIENTE ANGULAR

$$y - y_1 = m(x - x_1)$$

ÁLGEBRA

EXPOENTES E RADICAIS

$$a^m a^n = a^{m+n} \quad a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

$$(a^m)^n = a^{mn} \quad \sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$(ab)^n = a^n b^n \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$$\frac{a^m}{a^n} = a^{m-n} \quad a^{-n} = \frac{1}{a^n}$$

VALOR ABSOLUTO ($d > 0$)

$$|x| < d \quad \text{se e só se} \quad -d < x < d$$

$$|x| > d \quad \text{se e só se} \quad x > d \text{ ou } x < -d$$

$$|a+b| \leq |a| + |b| \quad (\text{desigualdade do triângulo})$$

$$-|a| \leq a \leq |a|$$

DESIGUALDADES

$$\text{Se } a > b \text{ e } b > c, \text{ então } a > c$$

$$\text{Se } a > b, \text{ então } a + c > b + c$$

$$\text{Se } a > b \text{ e } c > 0, \text{ então } ac > bc$$

$$\text{Se } a > b \text{ e } c < 0, \text{ então } ac < bc$$

FÓRMULA QUADRÁTICA

Se $a \neq 0$, as raízes de $ax^2 + bx + c = 0$ são

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

LOGARITMOS

$$y = \log_a x \quad \text{significa } a^y = x \quad \log_a 1 = 0$$

$$\log_a xy = \log_a x + \log_a y \quad \log_a a = 1$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y \quad \log x = \log_{10} x$$

$$\log_a x^r = r \log_a x \quad \ln x = \log_e x$$

TEOREMA BINOMIAL

$$(x+y)^n = x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \dots + \binom{n}{k} x^{n-k} y^k + \dots + y^n,$$

$$\text{onde } \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

PROPIEDADES DE LAPLACE

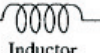
1	$\mathcal{L}[Af(t)] = AF(s)$
2	$\mathcal{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
3	$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$
4	$\mathcal{L}\left[\frac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$
5	$\mathcal{L}\left[\frac{d^n}{dt^n}f(t)\right] = s^nF(s) - \sum_{k=1}^n s^{n-k}f^{(k-1)}(0\pm)$ onde $f^{(k-1)}(t) = \frac{d^{k-1}}{dt^{k-1}}f(t)$
6	$\mathcal{L}\left[\int f(t) dt\right] = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t) dt\right]_{t=0\pm}$
7	$\mathcal{L}\left[\int \dots \int f(t)(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^n \frac{1}{s^{n-k+1}}\left[\int \dots \int f(t)(dt)^k\right]_{t=0\pm}$
8	$\mathcal{L}\left[\int_0^t f(t) dt\right] = \frac{F(s)}{s}$
9	$\int_0^\infty f(t) dt = \lim_{s \rightarrow 0} F(s) \quad \text{se } \int_0^\infty f(t) dt \text{ existir}$
10	$\mathcal{L}[e^{-at}f(t)] = F(s + a)$
11	$\mathcal{L}[f(t - \alpha)1(t - \alpha)] = e^{-as}F(s) \quad \alpha \geq 0$
12	$\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$

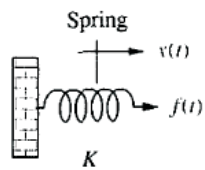
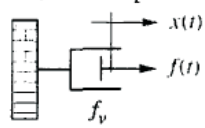
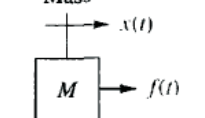
13	$\mathcal{L}[t^2f(t)] = \frac{d^2}{ds^2}F(s)$
14	$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}F(s) \quad (n = 1, 2, 3, \dots)$
15	$\mathcal{L}\left[\frac{1}{t}f(t)\right] = \int_s^\infty F(s) ds \quad \text{se } \lim_{t \rightarrow 0} \frac{1}{t}f(t) \text{ existir}$
16	$\mathcal{L}\left[f\left(\frac{1}{a}\right)\right] = aF(as)$
17	$\mathcal{L}\left[\int_0^t f_1(t - \tau)f_2(\tau) d\tau\right] = F_1(s)F_2(s)$
18	$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(p)G(s - p) dp$

RELAÇÕES RECORRENTES SÉRIE FOURIER

- (1) $\cos(n\pi) = (-1)^n = \begin{cases} 1 & , \text{ se } n \text{ é par} \\ -1 & , \text{ se } n \text{ é ímpar} \end{cases} , \forall n \in \mathbb{Z}.$
- (2) $\sin(n\pi) = 0, \forall n \in \mathbb{Z}.$
- (3) $\int x \sin(n\omega_0 x) dx = -\frac{x}{n\omega_0} \cos(n\omega_0 x) + \frac{1}{n^2 \omega_0^2} \sin(n\omega_0 x).$
- (4) $\int x \cos(n\omega_0 x) dx = \frac{x}{n\omega_0} \sin(n\omega_0 x) + \frac{1}{n^2 \omega_0^2} \cos(n\omega_0 x).$
- (5) $\int x e^{-in\omega_0 x} dx = \left(\frac{ix}{n\omega_0} + \frac{1}{n^2 \omega_0^2} \right) e^{-in\omega_0 x}.$
- (6) $\int x^2 \sin(n\omega_0 x) dx = -\frac{x^2}{n\omega_0} \cos(n\omega_0 x) + \frac{2x}{n^2 \omega_0^2} \sin(n\omega_0 x) + \frac{2}{n^3 \omega_0^3} \cos(n\omega_0 x).$
- (7) $\int x^2 \cos(n\omega_0 x) dx = \frac{x^2}{n\omega_0} \sin(n\omega_0 x) + \frac{2x}{n^2 \omega_0^2} \cos(n\omega_0 x) - \frac{2}{n^3 \omega_0^3} \sin(n\omega_0 x).$

MODELAGEM DE SISTEMAS EM LAPLACE

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C} \int_0^t i(\tau) d\tau$	$i(t) = C \frac{dv(t)}{dt}$	$v(t) = \frac{1}{C} q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R} v(t)$	$v(t) = R \frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L \frac{di(t)}{dt}$	$i(t) = \frac{1}{L} \int_0^t v(\tau) d\tau$	$v(t) = L \frac{d^2 q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
 Spring K	$f(t) = K \int_0^t v(\tau) d\tau$	$f(t) = Kx(t)$	K
 Viscous damper f_v	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
 Mass M	$f(t) = M \frac{dv(t)}{dt}$	$f(t) = M \frac{d^2 x(t)}{dt^2}$	Ms^2